EXPERIMENTAL OPTICS

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BY

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PREFACE

THIS laboratory manual contains the fundamental practical work in optics given to junior officers of the United States Navy taking the Ordnance Course at the U. S. Navy Postgraduate School, Annapolis, Maryland. Some of these officers will have duty as inspectors of military telescopic instruments to be purchased for use in the Navy. In order to perform such duty intelligently, a fundamental knowledge of geometrical optics is necessary.

The contribution made by Americans to the development of high-grade optical instruments is almost negligible compared to that made by Europeans. The chief reasons for this seem to be lack of interest, and failure to appreciate the importance of the subject of geometrical optics. There are very good reasons for supposing that the success of the Germans in the early part of the famous naval battle of Jutland was very largely due to superior optical equipment.

The U. S. Navy Postgraduate School at Annapolis has provided for the author a most unusual opportunity to develop a laboratory course in optics, which is primarily geometrical optics designed to equip the student with a working fundamental knowledge of telescopic optics. The fundamental ideas in many of the experiments in this laboratory manual are due to Mr. Max Zwillinger, Optical Engineer, Washington Navy Yard, Washington, D. C., and the author takes this opportunity to indicate his appreciation of Mr. Zwillinger's most valuable advice and encouragement in the development of this course of laboratory experiments. While this course of laboratory experiments was developed for students whose life work is in the Navy, the experiments are just as fundamental to any student of optics as they are to the students at the U. S. Navy Postgraduate School.

It is assumed that the problem of providing necessary equipment for performing the experiments in this manual will be solved by the instructor. Descriptions of such equipment have been

omitted but will be gladly furnished by the author upon request.

The author's thanks are due to the Bausch & Lomb Co., the Gaertner Scientific Co., and the Leeds & Northrup Co. for permission to use cuts of their apparatus.

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EXPERIMENTAL OPTICS

MISCELLANEOUS INTRODUCTORY NOTES

1. The Angular Scale Collimator Target.—For testing and making measurements on lenses and telescopic instruments, it is highly desirable to have at hand either a distant test target graduated in degrees or the equivalent of such a target.

If a distant target is to be employed, the form shown by Fig. 1 would be convenient. In order to understand fully how this target becomes direct reading, let us assume that it is the intention to have the radius 01^+ or 01^- of the inner circle subtend at the observing station an angle of 1 degree, the radius 02^+ or 02^- of the second circle an angle of 2 degrees, etc. Obviously, if the radius of the inner circle is to subtend at some observing station an angle of 1 degree, the length of this radius will be determined by the distance between the target and the observing station.

Fig. 1.

In order to show exactly how such a target would be made, let us make the necessary calculations.
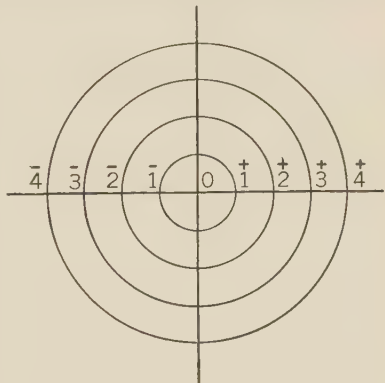
It will be necessary to select the distance between the target and the observing station. Call this distance 1000 meters. In Fig. 2, S represents the observing station, TT' represents the target similar to that shown in Fig. 1 with its plane perpendicular to the plane of the paper and its center at O. The distance SO has been made 1000 meters. 01^+ or 01^- represents the radius of the inner circle, and the problem is to make its physical length such that it subtends at S an angle of 1 degree.

The necessary physical length for radius 01^+ or 01^- is about 17.5 meters. In order for the radius 02^+ or 02^- to subtend at S an angle of 2 degrees, the required physical length for 02^+ or 02^- is about 35 meters. The radii corresponding to other angles can be readily calculated. The appearance of such a target is not difficult to imagine. It might consist of a large vertical wooden structure similar to a bill-board. The surface would be painted white, the circles of Fig. 1 and a vertical and horizontal diameter would be painted black.
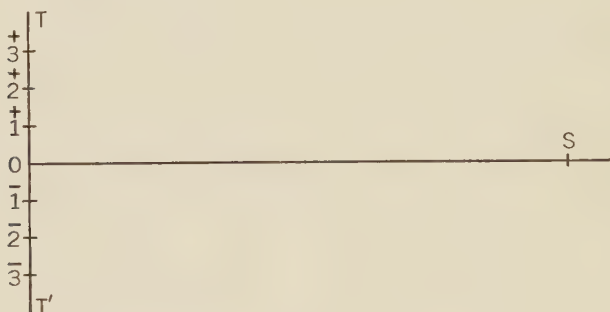
Fig. 2.

While such a target under certain conditions might be very useful, there are some serious objections to it, namely:

1. Because of its size the target must be outdoors.

2. Observations on such a target could be made only under favorable weather conditions.

3. A suitable site for such a target might be difficult to find.

4. Expense would be unnecessarily great.

5. Such a target would not be portable.

In view of the objections just mentioned, we will dismiss the idea of actually using such a target and proceed to the discussion of a suitable substitute or equivalent.

In Fig. 3, TT' represents a small illuminated target placed in the left-hand focal plane of lens L_c, called the collimator lens. All rays originating at any point in the target will emerge from the lens L_c parallel to each other. The courses of two rays from the center O of the target, two from point 1^+, and two from point 1^- are shown. Assume that distances 01^+ and 01^- in target TT' are equal. To an observer to the right of lens L_c the

emergent rays 1 and 2 appear to have their origin in a point
that is at infinite distance to the left of L_c and above the principal
axis.   Emergent rays 3 and 4 appear to have their origin in a
point that is on the principal axis at an infinite distance to the
left of L_c.   Emergent rays 5 and 6 appear to have their origin in
a point that is at an infinite distance to the left of L_c and below
the principal axis.

To an observer stationed to the right of L_c the visual effect of
placing a small illuminated target in the left-hand focal plane of
L_c is exactly the same as that produced if a large target placed at
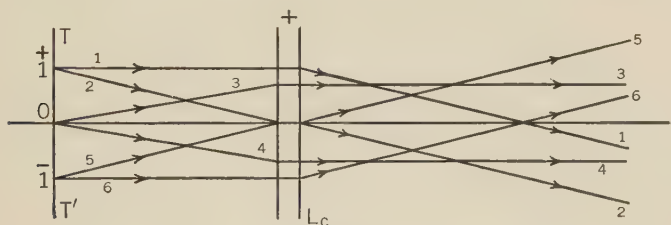
FIG. 3.

a very considerable ("practical infinity") distance to the left were
substituted for the small illuminated target and the lens L_c.

Let us assume that bundle 1–2 and bundle 5–6 are incident
from the right and the angle between them is 2 degrees, or 0.0350
radian.   The parallel rays in bundle 1–2 intersect at a point 1^+
in the left focal plane of lens L_c, and the parallel rays in bundle
5–6 intersect at a point 1^- in the left focal plane of L_c.   The distance
y between points 1^+ and 1^- is given by the fundamental relation

$$y = f\theta$$

where f is the focal length of lens L_c, and θ is the angle in radians
between bundles 1–2 and 5–6 incident from the right.   Assume
that the focal length of lens L_c is 308 mm.   Then, when $\theta = 2$
degrees $= 0.0350$ radian

$$y = (308) (0.0350) = 10.78 \text{ mm.}$$

The distance between the two points 1^+ and 1^- is, therefore,
10.78 mm.

If now points 1^+ and 1^- are located in an illuminated target,
all rays diverging from 1^+ and incident on lens L_c will emerge from

L_c to form a parallel bundle 1–2, and all rays diverging from 1^-
and incident on lens L_c will emerge to form a second parallel
bundle 5–6. If the distance between points 1^+ and 1^- is 10.78 mm.,
and the focal length of lens L_c is 308 mm., the angle between the
emergent parallel bundles corresponding to points 1^+ and 1^- must
be 2 degrees.

The result to an observer to the right of lens L_c is exactly the
same as though he were viewing a distant target containing two
points 1^+ and 1^- that subtended at the observing station an
angle of 2 degrees. In the
illuminated target assume a
point O midway between 1^+
and 1^-. With this point O as
a center, describe a series of
concentric circles to produce a
target of the form shown in
Fig. 4. Since we have already
shown that the *diameter* $1^+ 01^-$
is to correspond to an angle of
2 degrees, the *radius* 01^+ or
01^- corresponds to an angle
of 1 degree. The radius of the
second circle must correspond to
an angle of 2 degrees, the radius of the third circle must cor-
respond to an angle of 3 degrees, etc.
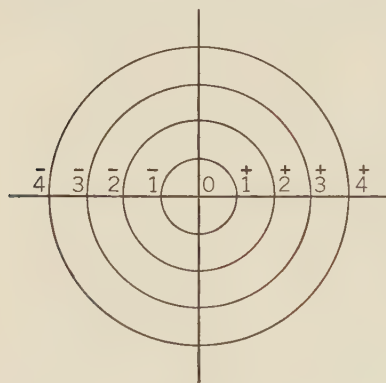
Fig. 4.

For a lens of focal length 308 mm., the radii of the circles in
the target shown in Fig. 4 corresponding to the different angles
are given in Table I.

TABLE I

Radius Designation	Length of Radius in mm.	Angle in Degrees
01^+ or 01^-	5.39	1
02^+ or 02^-	10.78	2
03^+ or 03^-	16.17	3
04^+ or 04^-	21.56	4

Assume the target described and specified above placed at
the left end of a brass tube of circular cross-section in the left
focal plane of a lens of focal length 308 mm., this lens being

situated at the right end of the brass tube. The effect produced on the retina of an observer who looks into the lens end (right end) of this tube is exactly the same as though he were looking at a distant target consisting of four concentric circles, the diameter of the largest circle subtending an angle of 8 degrees. If a telescope is pointed toward the lens end of the tube, the observer at the ocular end of the telescope may see an image of only a portion of the target, say, 4 degrees. This would mean that the *true field* of the telescope is 4 degrees.

The system just described and specified, consisting of a circularly graduated target, placed in the focal plane of a lens at one end of a brass tube, with the lens at other end of the same tube, will be referred to as an "angular scale collimator target."

If a converging lens system is placed opposite the lens end of the arrangement just described, so that its principal axis coincides with that of the collimator lens L_c, a real image of the graduated target will be formed in the rear focal plane of the converging lens system. In order to determine the focal length of the converging lens system, we may proceed as follows:

1. By any suitable means (probably micrometer microscope) measure the length y of the radius corresponding to any angle θ in the target.

2. Calculate the focal length f from the relation:

$$y = f\theta.$$

The idea just suggested forms the basis of a precise and convenient method for measuring the focal length of any type of converging lens system.

2. The "One to One" or Direct-Reading Micrometer Microscope.—One of the most convenient instruments for making measurements in geometrical optics is the so-called "one to one" micrometer microscope. The principle of this instrument can be made clear by reference to Fig. 5. T is a small target, L is the objective lens of the microscope, and I is the image of T. T, L, and I are arranged for unit magnification, that is, T and its image I are equal in size. This means that the distance from T to L and the distance from L to I are equal to each other and to $2f$, where f is the focal length of L. E is the ocular of the microscope. L and E are fixed in position with respect to each other, so that

the image I lies in the left focal plane of E. A direct-reading linear scale is also placed in the image plane of I. An observer looking into the ocular E will see in the field of view an image of target T and also an image of the linear scale. It should now be obvious to the reader that the length of any object placed at the position of T can be measured by noting the scale reading as seen in the field of view of the instrument, provided, of course, that object length is not greater than total range of scale.
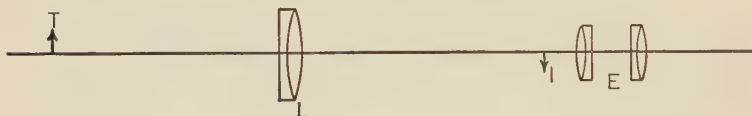
Fig. 5.

In order for such an instrument to read correctly, it is important that the ocular be kept in the proper fixed position with respect to L. If the ocular is moved from this position, a multiplying factor must be applied to the observed reading.

When the objective L and ocular E are once put in their proper relative positions, then the object under observation must be placed at a distance in front of the objective equal to $2f$, where f is the focal length of the objective.

3. A Collection of Important Formulae Used in Elementary Geometrical Optics.

1. Spherometer Formula:

$$R = \frac{r^2 + h^2}{2h}$$

where R is radius of curvature being measured;

r is radius of base circle of spherometer;

h is "vertical shift" or difference in the two spherometer scale readings.

2. Universal Focal-Length Formula for Simple Spherical Lenses (to be used with sign convention):

$$D = \frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{(n - 1)^2 d}{n R_1 R_2}$$

where D is dioptric power;

f is focal length;

n is index of refraction;

R_1 and R_2 are radii of curvature of first and second surfaces respectively;

d is "thickness" of lens (measured along principal axis).

In using this formula it is necessary to employ a sign convention for the quantities R_1 and R_2. This sign convention may be stated as follows:

(a) Assume a direction for the incident light.

(b) The radius of curvature of the surface of the lens at which the light is incident must be called R_1. The radius of curvature of the surface of the lens at which the light emerges must be called R_2.

(c) Measure R_1 and R_2 *from* the poles or vertices. These quantities are to be given a positive sign if they extend in the direction in which the light is assumed to travel, and a minus sign if they extend in the opposite direction.

(d) The idea in (c) may be expressed in another way as follows: Imagine a person traveling from a pole or vertex of the lens toward the center of curvature of the corresponding surface. If he is advancing in the direction in which the light is assumed to travel, the radius of curvature of the surface in question is to be given a positive sign; if in the opposite direction, the sign is to be negative.

If, as a result of employing the convention just stated, f is a positive quantity, the lens is converging and can be used to form real images; if f is a negative quantity, the lens is diverging and cannot be used by itself to form real images.

3. Universal p, q, f Lens Formula (to be used with sign convention):

$$\frac{1}{q} = \frac{1}{f} + \frac{1}{p}$$

where p is object distance;

q is image distance;

f is focal length.

In using this formula it is necessary to employ a sign convention for the quantities p, q, and f. This sign convention may be stated as follows:

Measure all values of p, q, and f *from* the corresponding equivalent points. These quantities are to be given a positive sign if they extend in the direction in which the light is assumed to travel, and a negative sign if they extend in the opposite direction.

4. Universal Principal-Point Formulae for Simple Lenses (to be used with sign convention):

$$h = \frac{(n-1)df}{nR_2} \qquad (a)$$

$$h' = -\frac{(n-1)df}{nR_1} \qquad (b)$$

where h is the distance from the first pole or vertex to the first principal point;

h' is the distance from the second pole or vertex to the second principal point.

In using these principal-point formulae it is necessary to employ a sign convention, which may be stated as follows:

The signs of R_1, R_2, and f are to be determined by the sign conventions employed for the Universal Focal-Length Formula, that is, Formula 2 above.

If, as a result of employing the convention just stated, h is a positive quantity, the first principal point is on that side of the first pole or vertex from which the light is incident. A negative sign means the reverse.

If, as a result of employing the sign convention of Formula 2, h' is a negative quantity, the second principal point is on that side of the second pole or vertex from which the light is incident. A positive sign means the reverse.

5. Effective Focal Length of a System of Thin Simple Lenses in Contact:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4} + \cdots$$

where F is the effective focal length of the system;

f_1, f_2, f_3, f_4, etc., are the focal lengths of the component simple lenses.

Due regard must be paid to the algebraic signs of f_1, f_2, f_3, f_4, etc.

6. Effective Focal Length f_s of a Combination of Two Lens Systems of Focal Lengths f_1 and f_2 Separated in Air by a Distance d:

$$f_s = \frac{f_1 f_2}{f_1 + f_2 - d}.$$

7. "Back Focus," $B.F.$, of a Combination of Two Lens Systems of Focal Lengths f_1 and f_2 Separated in Air by a Distance d:

$$B.F. = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}.$$

8. "Front Focus," $F.F.$, of a Combination of Two Lens Systems of Focal Lengths f_1 and f_2 Separated in Air by a Distance d:

$$F.F. = -\frac{f_1(f_2 - d)}{f_1 + f_2 - d}.$$

9. Universal Formulae for Refraction in the Paraxial Region at a Spherical Surface (to be used with sign convention):

$$\frac{1}{q} = \frac{n_1 - n_0}{n_1 r_0} + \frac{n_0}{n_1 p}.$$

$$f_1 = \frac{n_1 r_0}{n_1 - n_0}.$$

$$f_0 = -\frac{n_0 r_0}{n_1 - n_0}.$$

Employ the sign conventions, Formulae 2 and 3.

10. Universal Spherical-Mirror Formula ("special sign convention"):

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

where p is object distance;

q is image distance;

f is focal length.

In using this formula it is necessary to employ a "special sign convention," which may be stated as follows:

Measure all values of p, q, and f *from* the pole of the mirror. These quantities are to be given a negative sign if they extend in

the direction in which the incident light is assumed to travel, and a positive sign if they extend in the opposite direction. Another way of stating the same convention is that distances measured on the side of the mirror from which the light is incident are positive, and distances on the opposite side are negative.

11. Lens-Image-Magnification Formula:

$$\frac{OA}{IB} = \frac{p}{q}$$

where OA is object size;

IB is image size;

p is object distance;

q is image distance.

12. Size of Lens Image of an Infinitely Distant Object:

$$y = f\theta$$

where y is size of image of distant object formed in second focal plane of lens;

f is first focal distance of lens system that forms the image;

θ is actual angle subtended by the object at the lens system.

4. The Auto-Collimating Measuring Telescope.—The principle of the auto-collimating measuring telescope is explained in the discussion of the method in Experiment 48. The optical arrangement is shown in Fig. 81.

SECTION I

INDEX OF REFRACTION AND SPECTROMETRY
(Visible Spectrum)

EXPERIMENT 1

Index of Refraction of Glass and Water by Traveling Microscope

Procedure:

1. Sprinkle some lycopodium powder on the plane horizontal surface $H_1 H_2$, Fig. 6. Focus the microscope M, Fig. 6, on the powder placed on the $H_1 H_2$ plane, and read the position of the microscope on the graduated track T.

2. Place the approximately plane parallel glass specimen $ABCD$ over the powder and move the microscope upward along its graduated track so that the lycopodium powder is again in focus. The lycopodium powder has *apparently* moved up to the $I_1 I_2$ plane in Fig. 6. Read this position of the traveling microscope on its graduated track.

3. Sprinkle some lycopodium powder on top of the glass specimen $ABCD$ and move the microscope upward until the lycopodium powder just mentioned is in focus, that is, the microscope is in sharp focus for the $J_1 J_2$ plane. Read this position of the microscope.

4. The distance d_t in Fig. 6, i.e., the distance between the $H_1 H_2$ plane and the $J_1 J_2$ plane is the true thickness of the glass specimen. The distance d_A, i.e., the distance between the $I_1 I_2$ plane and the $J_1 J_2$ plane, is the *apparent* thickness. It can be proved that

$$n = \frac{d_t}{d_A} \tag{1}$$

where n is the index of refraction of the glass specimen.

The difference between the first and third track positions of

the microscope mentioned above is the true thickness, or d_t in equation (1). The difference between the second and third track positions is the apparent thickness, or d_A in equation (1).
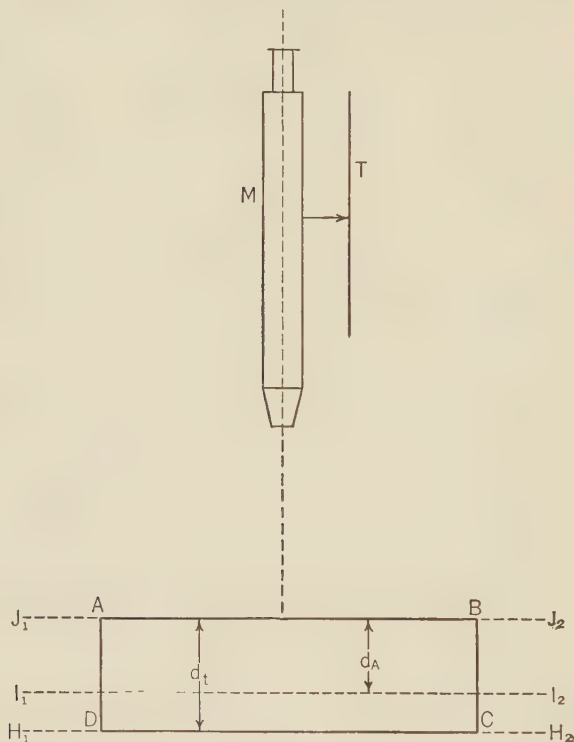
FIG. 6.

Results Required:

Index of refraction of each glass sample and of water or other liquids supplied.

EXPERIMENT 2

Determination of Index of Refraction of the Glass of a Thin Converging Lens by " Lens Formula " Method

Procedure:

1. Measure the radii of curvature R_1 and R_2 of the two lens surfaces by spherometer.

2. Measure focal length f of lens by two methods to be specified by instructor.

3. Calculate the index of refraction n from the thin-lens formula, which reads as follows:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{2}$$

R_1 and R_2 were measured in step 1. f was measured in step 2. Hence n can be calculated. For f use the average obtained by the two methods employed in step 2.

Precaution: The standard sign convention must be followed in using equation (2).

EXPERIMENT 3

Measurement of Index of Refraction of a Glass Prism by Use of the Spectrometer

Theory:

The spectrometer consists, as shown in Fig. 7, essentially of a divided circle LMN about the axis of which a collimator C and a
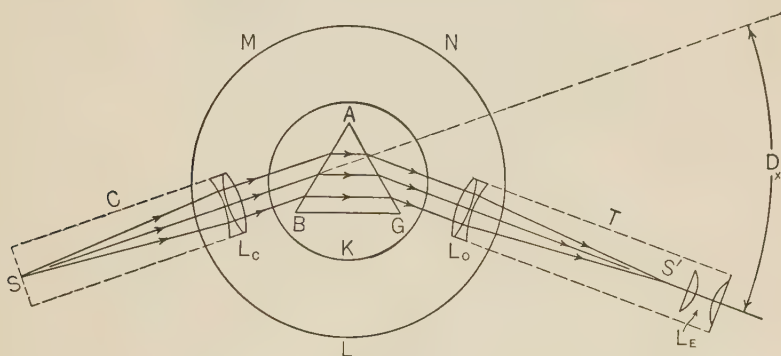
FIG. 7.

telescope T can rotate. The collimator is a tube at one end of which there is a well-corrected converging lens L_c, and at the other end of which there is a vertical slit S placed in the focal plane of L_c. Assume slit S to be illuminated by monochromatic light. Those rays which diverge from S and strike the collimator lens L_c will emerge from L_c as a parallel beam and will be incident on face AB of the prism ABG which rests on the prism table K. The table K can be rotated about the axis of the divided circle independently of C and T. The rays incident on face AB of the prism ABG are refracted at face AB and again at face AG and

emerge as a parallel beam and enter the objective lens L_o of the telescope T set for infinity. These rays now converge to form a real image of slit S at S' in the right-hand focal plane of L_o, and this image is observed by looking into the ocular lens system L_E of the telescope. The beam of parallel rays emerging from the collimator C has been deviated by the prism ABG by an amount represented by the angle D_x in Fig. 7.

Figure 8 shows a typical laboratory spectrometer.

In order to be able to make accurate measurements with the
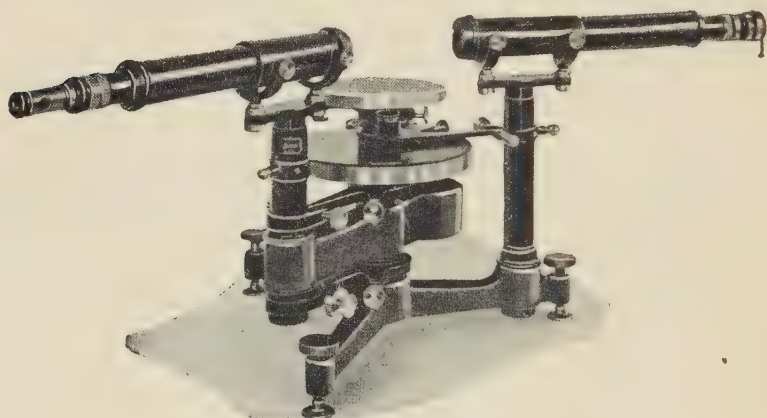
Fig. 8.—Laboratory Spectrometer.

spectrometer, it is important that certain adjustments be made, as follows:

1. Telescope T set for infinity, i.e., focused for incident parallel rays.

2. Slit S placed in focal plane of collimator lens L_c.

3. The principal axes of the telescope and collimator coincident and intersecting the axis of rotation of the instrument at right angles.

4. The prism faces AB and AG in Fig. 7 both parallel to the axis of rotation of the instrument.

We will now proceed to show how the index of refraction of a glass prism can be determined by the use of the spectrometer.

Let ABC, Fig. 9, represent the section of a glass prism, and let PQ be a ray of monochromatic light incident on the face AB at Q. Due to refraction at Q and at R, the general direction of the ray will be $PQRS$. Usually, two of the faces AB and AC are polished

and the third face BC is unpolished or ground.   The edge passing through A perpendicular to the plane of the paper or the edge in which the polished faces meet, is called the "refracting edge" of the prism and the angle A is called the "refracting angle" of the prism.   The angle UTR by which the incident ray has been bent out of its original direction by the prism is called the "angle of deviation."

A change in the angle of incidence of ray PQ on face AB will
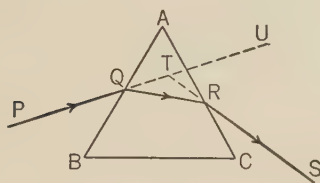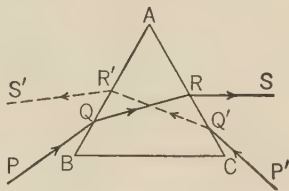
Fig. 9.   Fig. 10.

cause a change in the angle of deviation.   If the angle of incidence is equal to the angle of emergence, that is, if $AQ = AR$, the angle of deviation is a minimum and the prism is said to be in the position of minimum deviation.   That the latter statement is true can be proved as follows: If the deviation of any other ray $PQRS$, Fig. 10, is a minimum, then the deviation of a ray $P'Q'R'S'$ which passes through the prism in the symmetrically opposite direction, that is, so that $AQ = AQ'$ and $AR = AR'$, is the same as the deviation of $PQRS$.   This means that there would be two angles of incidence that would give minimum deviation, which is contrary to experimental fact.
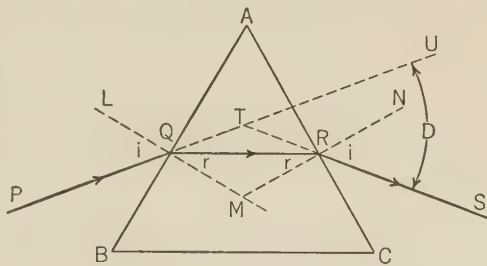
Fig. 11.

Consequently, the deviation of $PQRS$ is not a minimum, which means that the angle of deviation is a minimum for the case where the path of the ray is such that $AQ = AR$, as shown in Fig. 11. Figure 11 represents the condition for minimum deviation.

In Fig. 11, PQ is a ray incident at point Q of face AB of prism ABC. $PQRS$ represents the complete path of the ray. LM and NM are normals at the points of incidence and emergence, respec-

tively, of ray $PQRS$.   Let $PQL = SRN = i$, $RQM = QRM = r$, and the angle of minimum deviation $UTR = D$.   Then

$$D = TQR + TRQ \left. \begin{array}{l} \\ = TQM - RQM + TRM - QRM \\ = 2(i - r) \end{array} \right\} \quad (3)$$

Also

$$AQR + ARQ = \pi - A$$

and

$$AQR + ARQ = AQM - RQM + ARM - QRM$$

$$= \pi - 2r$$

Hence

$$r = \frac{A}{2} \qquad (4)$$

Substituting (4) in (3)

$$D = 2 \left[i - \frac{A}{2} \right]$$

or

$$i = \frac{A + D}{2}$$

But

$$n = \frac{\sin i}{\sin r}$$

Hence

$$n = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}} \qquad (5)$$

Equation (5) is the fundamental equation employed in measuring the index of refraction by the spectrometer method.   It is obvious that the method necessitates measuring the two quantities A and D, i.e., the refracting angle of the prism and the angle of minimum deviation.

The facts brought out in the preceding paragraphs can be verified by reference to Fig. 12.   The curves in this figure apply to a 60-degree prism having an index of refraction for sodium light equal to 1.500.   The curve labeled "Angle of Deviation Curve"

is plotted with angles of incidence on the x axis and corresponding angles of deviation on the y axis.   This curve shows that the angle of deviation has a minimum value of about 37.1 degrees when the angle of incidence is about 48.6 degrees.   The curve labeled "Angle of Emergence Curve" is plotted with angles of incidence on the x axis and angles of emergence on the y axis.   Inspection of the two curves shows that the angle of emergence is equal to the angle of incidence (both about 48.6 degrees) when the angle of
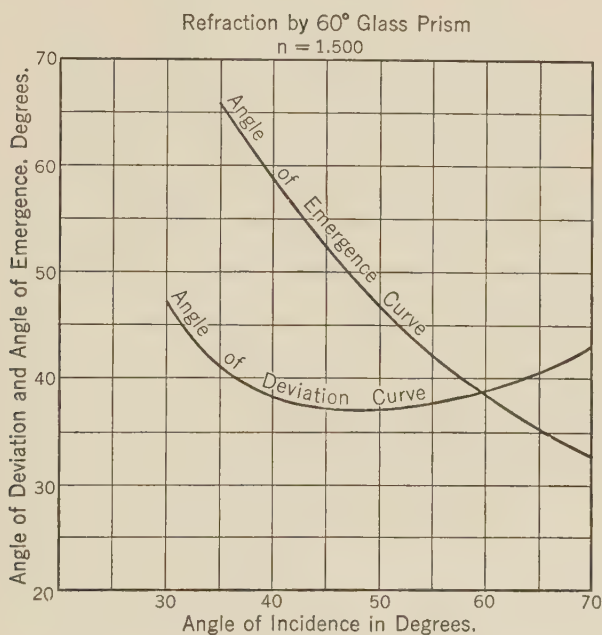
Fig. 12.

deviation has its minimum value of about 37.1 degrees.   This is another way of saying that the angle of deviation is a minimum for the case where the path of the ray is such that $AQ = AR$ in Fig. 11.

If the spectrometer telescope is pointed in the direction RS in Fig. 11, rotation of the prism table continuously in one direction will cause the image to move from one side of the field to the intersection of the cross-wires and then back, that is, the motion of the image reverses at the intersection of the cross-wires.   The

angle D can, therefore, be determined by reading the angular setting of the telescope for the condition described in the previous sentence and the angular setting for the undeviated case where the prism is removed. The difference between these two angular settings is D.

The refracting angle A may be measured as follows: Assume the spectrometer system so arranged, Fig. 13, that slit S is in the focal plane of the collimator lens L. All rays emerging from the collimator lens L will be parallel to each other. Consider two of these rays PQ and YZ incident on faces AB and AC respectively. These rays are reflected in the directions QM and ZN. If the prism is kept fixed and the telescope pointed in directions MQ and NZ, images of the slit will be seen. Call the angular difference between the setting for the MQ direction and that for the NZ direction a:
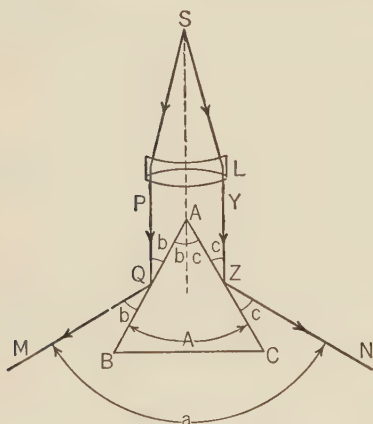
FIG. 13.

$$a = b + A + c$$

$$b + c = A$$

Hence

$$a = 2A$$

or

$$A = \frac{a}{2} \tag{6}$$

The procedure for measuring A consists in determining one-half the angle between the telescope position when set on the image due to light reflected from the left polished face in Fig. 13 and the telescope position when set on the image due to light reflected from the right polished face.

Results Required:

To be specified at time of experiment.

EXPERIMENT 4

Determination of Index of Refraction.
Prism-Spectrometer Method

Method:

The index of refraction is to be obtained from the formula

$$n = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}} \tag{7}$$

where A is the "refracting angle" of the prism, D is the "angle of minimum deviation," and n is the index of refraction.

1. Measurement of Refracting Angle of Prism

Determine the angular setting of the telescope for light reflected from: (a) the left face, and (b) the right face. Half the angle between these two settings represents the refracting angle A of the prism.

2. Measurement of Angle of Minimum Deviation

Illuminate the slit with sodium light, wave length from 5890 to 5896 Ångströms, and so place the prism that the sodium spectrum is visible in the telescope. The image sought in this case is formed by refracted light. When the image of the slit has been found in the telescope, adjust so that the image is visible in the field well to one side of the cross-wires. For the proper setting of the telescope, rotation of the prism continuously in one direction will cause the image to move to the cross-wires and then back again.

The angular setting of the telescope is to be read: (a) when the "turning point" is at the cross-wires, and (b) when the light from the slit comes directly through in a straight line, that is, with prism removed. The angle between the (a) and (b) settings is the minimum angle of deviation of the prism in question for sodium light.

In a similar manner measure the angle of minimum deviation for about eight prominent lines in the helium spectrum and for four prominent lines in the mercury-vapor spectrum.

Determine the wave length of the light for each spectral line employed by means of the constant-deviation wave-length spectrometer.

Results Required:

Calculated value of index of refraction for each wave length Curve with n values as ordinates and wave lengths as abscissae.

EXPERIMENT 5

Determination of Index of Refraction by Total-Reflection Methods

Theory:

It is often desirable to measure accurately the index of refraction of a substance which cannot conveniently be made into the form of a prism. Considerable time and expense may be involved in preparing a suitable prism for the spectrometer method, and in addition the solid or liquid must be transparent to be measured.

In methods employing total reflection, a single drop of a liquid is sufficient, and for a solid a small specimen with only one polished face is satisfactory; furthermore, these methods are applicable to determination of indices of refraction of solids or liquids which are imperfectly transparent, such as butter, milk, and lubricating oils.
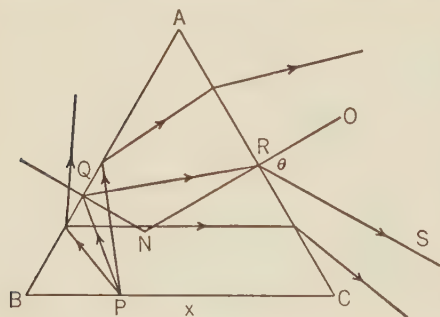
Fig. 14.

Let ABC, Fig. 14, be a glass prism, face BC being ground and faces AB and AC polished. Assume a sodium flame placed at some position X so as to illuminate the ground face BC. Consider any point P on this illuminated ground face. Rays will diverge from P in all directions. Let n be the absolute index of refraction of the prism and assume face AB covered with a liquid of absolute index of refraction n_1. If the angle of incidence i_c for ray PQ is such that $\sin i_c = n_1/n$, the angle of refraction is equal to 90 degrees and the refracted ray grazes the face BA and emerges in the direction QA. If the angle of incidence has a value i greater

than i_c the ray is totally reflected back into the prism and the intensity of the reflected ray is equal to that of the incident ray. If the angle of incidence is less than i_c, some of the light is refracted and some is reflected, and consequently the intensity of the reflected ray is less than that of the incident ray.

What has been said about rays from P will be true for rays from all other luminous points on BC. Consequently, if a telescope is focused for parallel light and pointed in the direction SR the field of view will be divided into two parts by a line corresponding to the direction SR. One side of this line is illuminated by light which has been totally reflected, and is consequently brighter than the other side. In making measurements by this total reflection method the intersection of the telescope cross-wires is set on the line of separation referred to above. The difference in the brightness on the two sides of the dividing line is usually not very great, and consequently some difficulty may be experienced in making a setting.

A modification of the optical arrangement just described will overcome the difficulty mentioned in the preceding paragraph. Assume the face BC covered with black paper and the sodium flame placed in the plane of AB and nearer B, so that the rays from the flame now fall on face BA at grazing incidence and at angles near grazing incidence. The ray at grazing incidence travels through the prism after refraction in the direction QR. The other rays make smaller angles of refraction with the normal QN. Hence if the telescope is pointed in the direction SR, the field on one side of the line of separation is bright and on the other side is dark. It is much easier to make a telescope setting for this condition than for the one described in the preceding paragraph.

In Fig. 14 assume a medium of refractive index n_1 in contact with face AB, and air in contact with face AC. Call the index of refraction of the prism n. Also designate PQN and RQN by i.

$$A + RNQ = \pi$$

or

$$RNQ + NRQ + i = \pi$$

where ORN is normal to AC at R

$$\therefore A = NRQ + i$$

or

$$i = A - NRQ.$$

Hence

$$n_1 = n \sin i$$
$$= n \sin (A - NRQ)$$
$$= n \sin A \cos NRQ - n \cos A \sin NRQ.$$

Applying Snell's Law at R,

$$\sin \theta = n \sin NRQ$$

whence

$$n_1 = \sin A \sqrt{n^2 - \sin^2 \theta} - \cos A \sin \theta. \quad (8)$$

For the special case where air is in contact with face AB, $n_1 = 1$, and equation (8) reduces to

$$n^2 = 1 + \left(\frac{\sin \theta + \cos A}{\sin A}\right)^2 \quad (9)$$

Equation (9) forms the basis of a total-reflection method for determining the index of refraction n of a glass prism.

For putting this method into practice there are several different types of so-called refractometers having scales from which the index of refraction can be read directly. The Abbe refractometer is one of the most widely used instruments of this type. Such instruments are usually calibrated with liquids of known index of refraction. They cannot themselves be used to determine the n or A of equation (8). Their arrangement is such that measurements of index of refraction can be made quickly and conveniently. An ordinary spectrometer can be adapted for use with this method and very satisfactory results are possible. The use of the spectrometer with this method offers the advantage that n and A in equation (8) can be determined, and hence n_1 can be obtained by the conscious use of the fundamental relation involved, namely, equation (8).

If an ordinary spectrometer is to be employed, the prism should have as high an index of refraction as possible, because its index of refraction must be higher than the indices of refraction to be measured. The first step is to measure n and A. Employ the prism-spectrometer method of Experiment 3 for this purpose. When n and A have been measured, clamp the prism table and fasten the prism to the prism table. If the index of refraction of a liquid is to be measured, place a drop or two of it on the polished

prism face AB in Fig. 15 and press a thin plate P_1P_2 of glass against this face of the prism. Capillary attraction keeps the plate in position and at the same time causes the drops to be spread in a thin layer between the plate and the face of the prism. The limiting direction RS of Fig. 14 is then determined by means of the telescope. The collimator is not used at all. If the light is to be incident internally, an image of a sodium flame should be formed on the ground face by means of a lens. The beam forming the image should strike the ground face so that its direction after refraction will be approximately PQ in Fig. 14. If the light is to be incident externally, the beam should be directed on the end RP_1 of the plate, as indicated in Fig. 15. Careful analysis will show that the beam cannot enter by the back of the plate P_1P_2 and

FIG. 15.

at the same time traverse the liquid in the required direction. If the end RP_1 of the plate is polished, it must be plane and parallel to the refracting edge A of the prism.
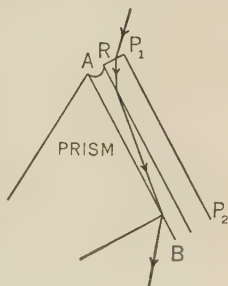
If the light is to be incident externally, it is much better to use, instead of the plate, another prism as similar to the first one as possible. In this case the direction of the incident beam is approximately parallel to the direction of emergence of the critical

FIG. 16.

ray, as shown in Fig. 16, so that to find the latter it is necessary only to rotate the prism table instead of moving the light source and telescope independently.
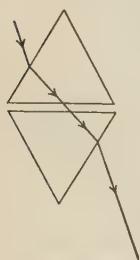
Bring the telescope to such a position that the cross-lines are exactly on the dividing line between the two unequally illuminated parts of the field. Read this angular position of the telescope. Now determine the angular position of the telescope such that its principal axis is normal to the face AC of the prism. The latter setting can be conveniently obtained by the aid of a Gauss eye-piece. The difference between the two angular positions of the telescope mentioned above represents the angle θ in equation (8).

n, A, and θ in equation (8) are now known, and hence n_1 can be calculated.

If the index of refraction of a solid is to be obtained by this method, a polished face of the specimen is placed against face AB, Fig. 14, with a drop of liquid of higher index of refraction than that of the solid between the specimen and the prism. The purpose of this liquid is to fill the interstices due to the lack of planeness of the two surfaces. The liquid commonly used for this purpose is monobrom naphthalene, the index of refraction of which for sodium light is 1.660.

Procedure:

Explained under "Theory" above.

Results Required:

Index of refraction of prism by use of equation (9). Indices of refraction of solid and liquid specimens supplied by use of equation (8).

EXPERIMENT 6

Spectrometer Adjustments

Theory:

Before proceeding to make measurements with the spectrometer it is necessary to see that certain conditions hold true, as follows:

1. The principal axes of the telescope and collimator must be perpendicular to the main axis of the instrument, i.e., the axis about which the telescope and prism table rotate.

2. The telescope must be focused for parallel incident rays, that is, for infinity.

3. The collimator must be focused for parallel emergent rays.

4. The prism must be adjusted so that the faces which include the angle to be measured are both parallel to the axis about which the telescope and table turn.

5. In certain spectrometers the telescope is mounted at the top of a rod which fits into a vertical stand tube made parallel to the main axis of the instrument. This means that the telescope can be raised or lowered in its stand tube and it can, of course, also be rotated about the axis of the stand tube. The same arrangement is provided for the collimator. In a spectrometer of this kind other conditions in addition to those already mentioned in steps 1 to 4 above must be satisfied. These conditions are as follows:

(a) The axes of telescope and collimator must pass through the main axis of the instrument.

(b) It must be possible to make the axes of the telescope and collimator coincident.

Procedure:

The five conditions mentioned under "Theory" are to be established in the order given.

1. If the instrument is provided with a Gauss eye-piece, that is, with an eye-piece having a side opening between the component lenses and a piece of clear glass inclined at 45 degrees to the axis, the following method may be employed to satisfy condition 1 above: When a small lamp is placed opposite the opening of the Gauss eye-piece light rays are reflected along the axis of the telescope. If, after emerging from the objective lens, these rays are reflected back into the telescope by means of a plane reflecting glass surface or by means of a plane mirror, a reflected image of the cross-wires will be seen alongside the direct image of the cross-wires by an observer looking into the eye end of the eye-piece. Assume a piece of optical glass having plane parallel sides mounted on the prism table with its faces approximately vertical. Adjust this piece of glass so that the image of the cross-wires formed by reflection at the surface of the glass coincides with the direct image. Rotate the telescope 180 degrees about the main axis of the instrument so that the light is now reflected back into the telescope from the second face of the piece of glass. If the reflected image of the cross-wires now coincides with the direct image, the axis of the telescope is perpendicular to the main axis of the instrument. If the reflected image does not coincide with the direct image, it must be brought half-way back by rotating the telescope about its horizontal axis, and half-way by adjusting the prism table which supports the plane parallel piece of glass. This adjustment must be repeated until the direct and reflected images of the cross-wires coincide for both positions of the telescope.

If the telescope is not fitted with a Gauss eye-piece, assume that the principal axis of the telescope is parallel to the telescope tube and proceed as follows: Place a level on top of the telescope tube. Turn the telescope so that it is parallel to the line joining two of the leveling screws of the instrument, and by adjusting

these screws bring the bubble of the level to the center. Now rotate the telescope 180 degrees about the main axis of the instrument, and if the bubble is no longer at the center, bring it half-way back by adjusting the leveling screws, and the other half-way by rotating the telescope about its horizontal axis. It is usually necessary to repeat the above procedure two or three times in order to keep the bubble exactly at the center as the telescope is rotated about the main axis of the instrument.

2. The telescope can be focused for parallel incident rays by focusing on any convenient distant object.

3. If the telescope is focused for parallel rays make the axes of the telescope and collimator approximately coincident, that is, point the telescope in the direction of the axis of the collimator. Now adjust the position of the slit tube in the main collimator tube so that the image of the slit seen in the field of view of the telescope is sharply defined. The collimator is now focused for parallel emergent rays.
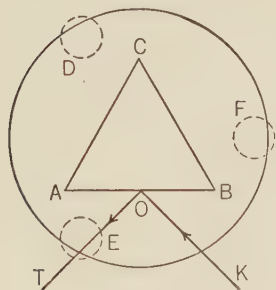
FIG. 17.

4. Mount the prism on the prism table so that the main axis of the instrument passes approximately through the center of the prism and so that one of the faces which includes the refracting angle A, Fig. 17, to be measured is perpendicular to a line joining two of the prism-table leveling screws, e.g., face AB in Fig. 17 is perpendicular to the line joining screws D and E. Set the telescope so that its axis makes an angle of about 90 degrees with the axis of the collimator and so that light from the collimator is reflected by prism face AB, Fig. 17, into the telescope. The path of the light is indicated in Fig. 17 by the ray KOT. Adjust the prism-table leveling screws so that the image of the middle of the slit coincides with the intersection of the cross-wires. Now rotate the prism table so that face AC reflects light from the collimator into the telescope, and adjust the leveling screw F so as to bring the image of the middle of the slit into coincidence with the cross-wires. Manipulation of screw F moves the face AB in its own plane, and consequently face AB will not be thrown out of adjustment while face AC is being adjusted by means of screw F. The prism adjustment is now complete.

5. The problem of making the adjustments to satisfy the conditions 5(a) and 5(b) will be left to the student.

EXPERIMENT 7

Determination of Dispersive Powers of Crown Glass, Flint Glass, Carbon Bisulphide, Alcohol, Turpentine, and Water

Procedure:

Using the prism-spectrometer method, obtain data for a curve coordinating on the y axis indices of refraction and on the x axis wave lengths for each of the substances supplied. Determine the index of refraction n in each case from the formula

$$n = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}}. \tag{10}$$

Results Required:

From the dispersion curve referred to under "Procedure" and the equation

$$d = \frac{1}{\nu} = \frac{n_F - n_c}{n_D - 1} \tag{11}$$

determine the value of d and ν for each substance supplied.

What conclusions can be drawn from the results obtained?

EXPERIMENT 8

Measurement of Index of Refraction of Solids and Liquids by Use of the Abbe Refractometer

Theory:

It was pointed out in Experiment 5 that the index of refraction of a transparent solid or liquid can be determined by measuring the critical angle of total reflection.

When using the Abbe refractometer it is not necessary to measure the value of the critical angle and then calculate the index by means of formula; the index can be read directly from a sector scale having intervals calculated for the constants of the glass used in the Abbe prisms. Figures 18 and 19 show the optical system and external view respectively of the instrument.

The border line of total reflection is observed by means of a telescope. For the purpose of bringing the border line of total reflection into coincidence with the intersection of the cross-hairs of the telescope, the high index prism B with which the substances to be measured are brought in contact is rotated about an axis perpendicular to the axis of the telescope. Attached to this
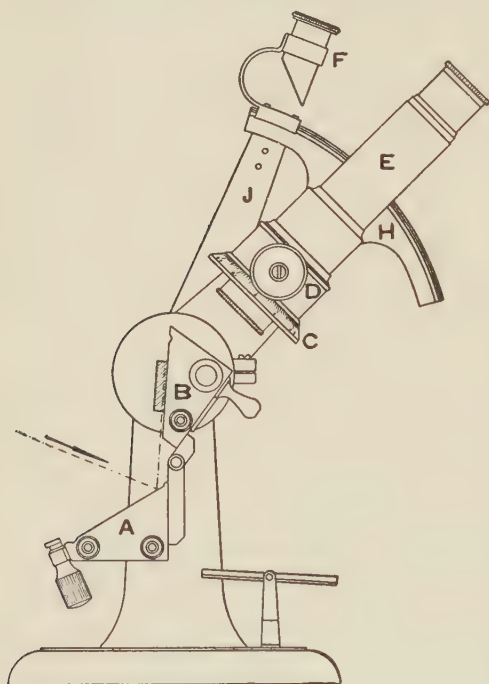
Fig. 18.

prism is an arm or alidade carrying an index, which moves along a sector scale as the prism is rotated and by means of which the index is read when the border line of total reflection has been brought to the intersection of the cross-hairs.

In order to obtain a thin film in measuring the index of liquids a second prism A is provided, which is so mounted that, when it has been clamped into position, its finely ground surface is located at a distance of about 0.1 mm. from the polished surface of prism B and parallel to it. The sole purpose of this prism is to lead light at "grazing incidence" into the liquid film, so that the light will strike the main prism face at "grazing incidence." In this position the ground surface of prism A becomes the light source for the rays which pass into the telescope.

When solids are measured, prism A is not utilized, but contact with prism B is obtained by means of some liquid. Under these conditions light enters the prism at "grazing incidence."

From the principle of the instrument it will be understood that

in all cases the index of refraction of substances measured must be less than that of the Abbe prism and that the index of any liquid used for the purpose of obtaining contact with the Abbe prism must be higher than that of the substance to be measured.

Figure 20 (*a*) shows the path of "grazing-incidence" light on the face of the main prism; Fig. 20 (*b*) shows the path of the light when measuring a solid; Fig. 20 (*c*) shows the path of the light when the liquid being measured is spread out as a thin film between the faces of the main prism *B* and the auxiliary prism *A*.

The Abbe refractometer consists essentially of four parts:

1. The telescope.
2. The Abbe prisms.
3. The sector.
4. The compensation prisms.

The telescope *E* consists of objective, eye-piece, and a cross-hair disc, which is placed in the focal plane of the objective.

Fig. 19.

The function of the telescope is to form an image of the border line of total reflection in the plane of the cross-hairs, whose inter-
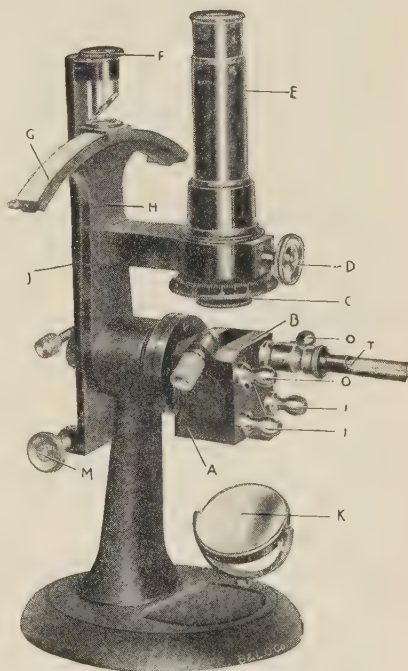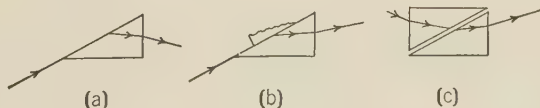
Fig. 20.

section provides a point with which the border line may be brought into coincidence under conditions assuring the greatest possible accuracy of setting.

The Abbe prisms, two similar flint-glass prisms of high refractive index, are cemented in hollow, water-jacketed mounts, so designed that when temperature control is desired water can be circulated around the prisms. The exposed surfaces of the upper prism B are highly polished, while the hypotenuse surface of the lower prism A, which serves solely for the purpose of illumination, is finely ground. The upper-prism mount, to which the lower mount is hinged, is rigidly attached to the alidade or index arm. When the lower mount is clamped into position against the upper, the hypotenuse surfaces of the prisms are separated by a space of 0.1 to 0.15 mm.

The sector H is a metal arm to which the telescope is rigidly connected. One end is attached to the upright of the base in such a manner that the whole sector may be rotated about an axis which coincides with the axis of rotation of the Abbe prism. This may be considered as the main axis of rotation of the instrument. The sector may be rotated within fixed limits to accommodate the various conditions of illumination at various points of the index scale. On the upper end is mounted a silver scale G, (Fig. 19), graduated directly in terms of refractive index for the D line for a temperature of 20° C.

An alidade or movable arm J, which may be rotated as a unit with the Abbe prisms, but independently with respect to the sector, around the main axis of the instrument, rests against the sector. The upper end of the alidade bears an index mark, which, when the arm is rotated, moves along the edge of the scale. A magnifier F is attached to the end of the alidade for reading the scale.

The compensation prisms are described under "Measurement of Dispersion."

Manipulation:

The method of manipulation of the instrument may be conveniently divided into two classes as follows:

 1. Measurement of liquids.

 2. Measurement of solids or plastic bodies.

MEASUREMENT OF LIQUIDS

Unclamp the prism mounts A and B. Clean the hypotenuse surfaces of the two Abbe prisms with alcohol, xylol, distilled

water, or some similar solvent, using for this purpose a clean, well-washed piece of linen or chamois skin. Every precaution should be taken to keep dirt or grit from coming in contact with, and being rubbed against, the polished surfaces of the Abbe prisms, as these prisms are made of relatively soft glass and are easily scratched. A camel's-hair brush will be found useful in removing dust particles that may have settled on the surfaces. The cloth or chamois skin used for cleaning the prism surface should be kept in some dust-proof receptacle when not in use.

Place a few drops of the liquid to be measured upon the surface of prism A and clamp the two mounts together. In order to obtain a sharp, bright image, enough liquid should be used to fill completely the space between the prisms when the mounts are clamped.

To facilitate measurements of rapidly evaporating fluids, a funnel opening has been provided at one corner of the prism mounts. It is only necessary to unclamp the prisms slightly, pour a few drops of liquid into the opening, and again tighten the prisms.

Adjust the mirror K so that the light source is reflected into prism A. Hold the telescope and sector stationary with one hand and move the alidade with the other from the position of lowest index (1.300) upward until a border line, separating a light and dark field, appears in the field of view of the telescope. This line is the border line of total reflection. Bring it into exact coincidence with the intersection of the cross-hairs in the telescope and read the refractive index by observing with the magnifier the position of the index line with reference to the sector scale.

Measurement of Solids

Solid specimens to be measured should have two polished surfaces at right angles to each other and intersecting in a sharp edge. Where an accurate result is not required, it is possible to obtain a suitable image by simply breaking the specimen at right angles to the one polished surface.

Turn the sector about the main axis until the instrument is in the position shown in Fig. 19. Place a drop of liquid of a higher index than the specimen on the larger polished surface of the specimen and place it in contact with the hypotenuse surface of

prism B, and as far away from the telescope as possible.   The liquid forms a thin film between the prism and the specimen, so that the latter is held in position by capillary attraction.   It is very important that the two surfaces in contact should be thoroughly clean and that the liquid used be free of any foreign substances that might scratch the prism surface.

The use of monobrom-naphthalene for indices under 1.64 and methylene-di-iodide for those above will be found to give satisfactory results.   The latter liquid should not be left uncorked because the air causes it to crystallize and turn dark.

The light reflected from the face of the mount A parallel to the hypotenuse face of the mount B enters the specimen at grazing incidence and is refracted through the liquid film and the prism. The measurement from this point is the same as in liquids.   Where monochromatic light (see "Measurement of Dispersion") is not used, care should be taken not to mistake the edge of a window frame, etc., for the border line.   A piece of white paper placed on the prism mount B will avoid any chance of an error.

Measurement of Dispersion

The Abbe refractometer was designed for use with white light, but for the sake of simplicity in the above description it has been assumed that sodium (monochromatic) light is used.

When daylight or any other source of white light is used, the border line of total reflection, instead of being clear and sharp, appears as a band of color.   This is due to the unequal refraction of the different wave lengths of which the light is composed.   In order to make it possible to use any type of light source, means must be provided to compensate for the dispersion of the light caused by the prisms and the substance being measured.

This is accomplished by placing two similar Amici prisms between the telescope objective and the double prism.   These prisms transmit sodium light ($589m\mu$) without deviation and are so mounted that they can be rotated simultaneously, but in opposite directions, about the telescope axis.   Thus the dispersion of these prisms can be made to vary in value from zero, when the bases of the prisms are opposite and parallel, to double the amount of dispersion of a single prism, when the bases are parallel and on the same side.   In order to get a colorless border line it is only

necessary to rotate the Amici prisms by means of the milled head D until there is obtained an equal but opposite disperison to that caused by the double Abbe prism in conjunction with the substance being measured.

When measuring substances having low dispersion, it is not always possible to obtain an absolutely colorless line, due to the secondary spectrum. Under these conditions the compensating prisms should be adjusted so that the bright half of the field appears colorless up to a very definite line, beyond which, toward the dark half of the field, will be visible a fringe of blue. In spite of this color disturbance accurate settings may be made by bringing the edge of the colorless half of the field to the intersection of the cross-hairs.

The amount of rotation of the compensating prisms with respect to one another forms a means for calculating the mean dispersion $(n_F - n_C)$ of a substance. The drum C, on which is engraved an arbitrary scale, rotates in conjunction with one of the prisms. The index on the front of the telescope tube provides a means for reading the drum. When the border line has been carefully achromatized, the drum reading, with reference to the index on the front of the telescope tube, is taken.

There are two positions of the compensation prism at which the border line will be colorless. These two positions are symmetrical with respect to the zero point of the drum and give the same drum readings. If measurements of the highest accuracy are desired, the means of settings made on both sides of the zero point should be taken.

A special chart is provided with each instrument, by means of which, when the index and the drum reading are known, the mean dispersion $(n_F - n_C)$ can be read directly. A portion of this chart is shown in Fig. 21.

Use of the Dispersion Chart

The horizontal lines on the chart represent the compensator or drum readings, while the vertical lines indicate the index for D. The curves represent the mean dispersion. The intersection of the two lines representing the drum reading and index is found. By interpolating from the values of the curves on either side of this point, the mean dispersion is read directly.
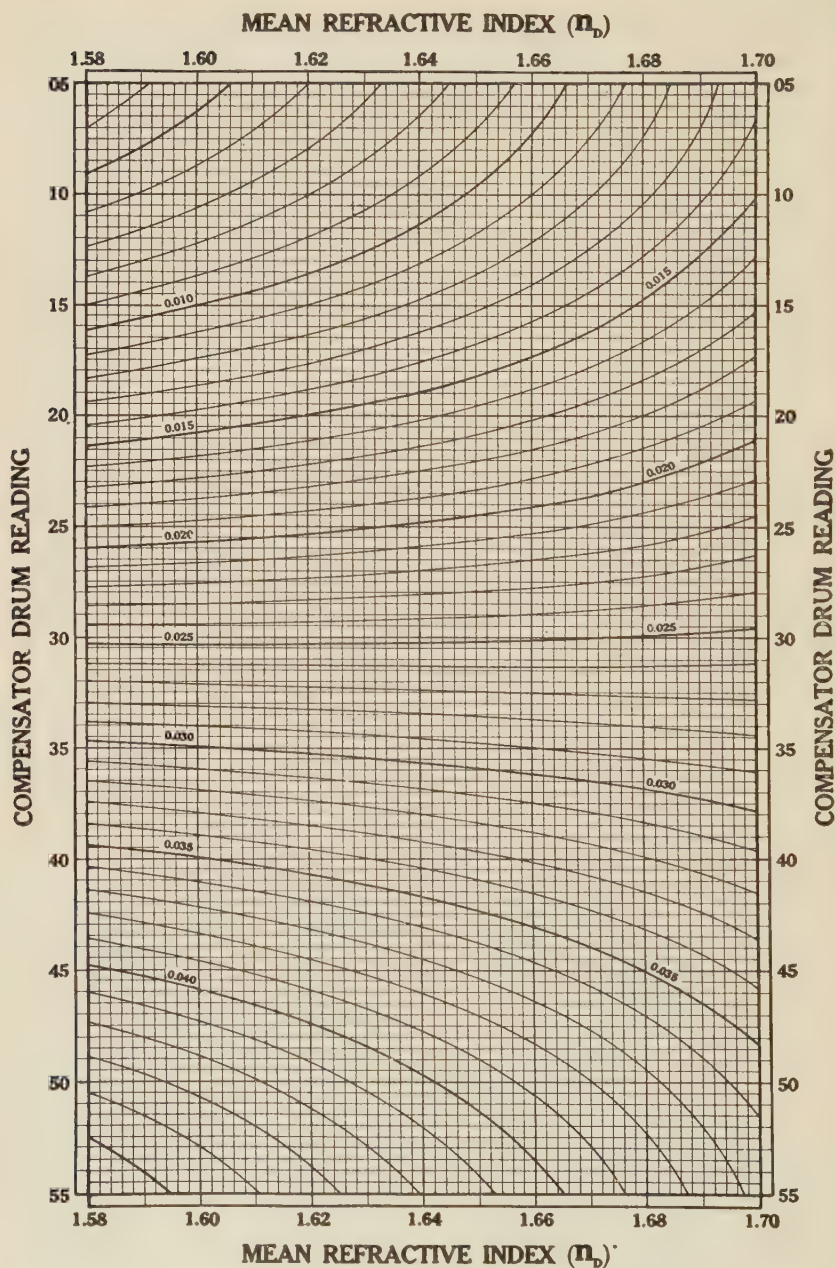
Fig. 21.

Example: Calcite (ordinary ray):

n_D 1.6585.   Drum reading, 14.7.

The point of intersection of the vertical line, representing 1.6585, and the horizontal line, representing 14.7, lies about halfway between the curves 0.013 and 0.014. Therefore, the mean dispersion ($n_F - n_C$) is 0.0135.

Results Required:

Index of refraction and dispersion of each solid and liquid supplied.

EXPERIMENT 9

Measurement of Wave Length by Use of the Constant-Deviation-Prism Spectrometer

Theory:

In Fig. 22, *ABCD* represents a prism known as a "constant-deviation" prism. While this prism is made in one piece, it may be considered to be equivalent to two "30, 60, 90 degree" prisms and one "45, 45, 90 degree" prism. Assume that this prism can be rotated through a small angle about an axis normal to the plane of the paper. For a certain angular setting of the prism the angle between any incident monochromatic ray and the corresponding emergent ray will be 90 degrees; for a second angular setting of the prism the angle of deviation is 90 degrees for a second monochromatic ray. As the prism is rotated, each monochro-

Fig. 22.

matic ray of the spectrum is incident in succession at 45 degrees on the face *BC* and consequently makes equal angles with the surface on entering and leaving the prism. As a result each monochromatic ray in turn is deviated exactly 90 degrees by the prism. The action of the prism on each monochromatic ray in turn is the same as that of the ordinary 60-degree prism at minimum deviation. If the telescope principal axis *MT*, Fig. 22, is kept fixed at an angle of 90 degrees with respect to the collimator
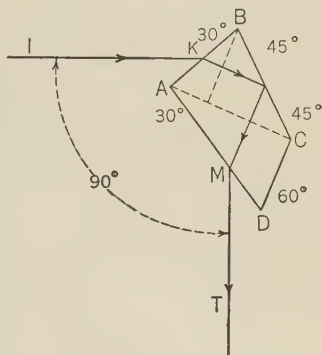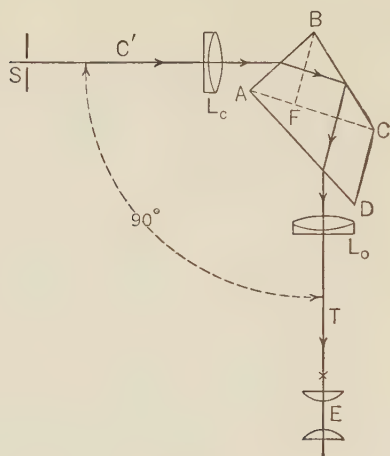
Fig. 23.
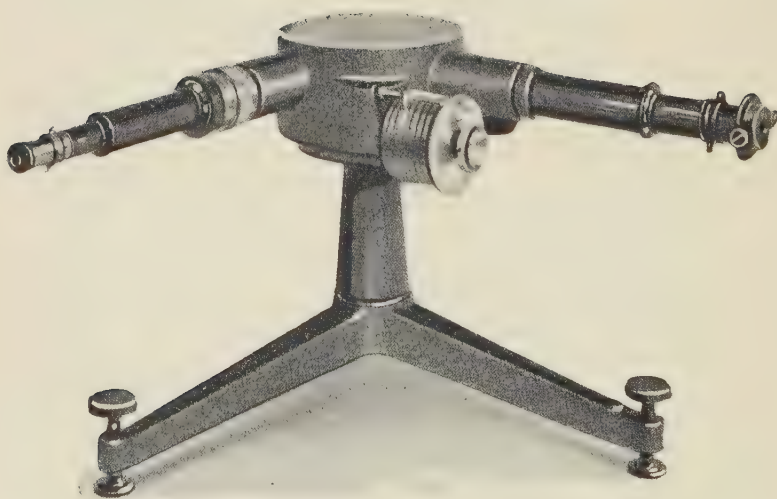
Fig. 24.—Constant Deviation Prism Spectrometer.

principal axis IK, then each monochromatic ray of the spectrum will be at minimum deviation as it passes the cross-wires of a telescope having principal axis MT.

Figure 23 shows the complete optical system of the constant-deviation-prism spectrometer. C' is the collimator with slit S at the principal focus of the collimator lens L_c. $ABCD$ is the constant-deviation prism, T is the telescope with objective lens L_o, ocular system E, and cross-lines at X. Telescope T and collimator C' are fixed at right angles to each other; the prism $ABCD$ is rotated by means of a drum graduated to read directly in wave length. The graduations on the drum move past an index. As a given spectral line appears at the intersection of the telescope cross-wires, its wave-length figure appears at the index.

Figure 24 shows a modern type of constant-deviation-prism spectrometer. This instrument is frequently called a wave-length spectrometer because wave lengths can be read directly from the scale.

Procedure:

Illuminate the slit of the constant-deviation-prism spectrometer with a helium tube. Measure the wave length of each spectral line. Repeat, using a mercury-vapor lamp and any other sources that may be specified.

Results Required:

To be specified at time of experiment.

SECTION II

FOCAL LENGTHS OF CONVERGING LENSES AND LENS SYSTEMS

EXPERIMENT 10

Focal Length of a Converging Lens. Conjugate-Foci Method

Method:

In Fig. 25, OA is an illuminated target, L_x is the test lens, and IB is a real inverted image of OA. OA must be placed to the left
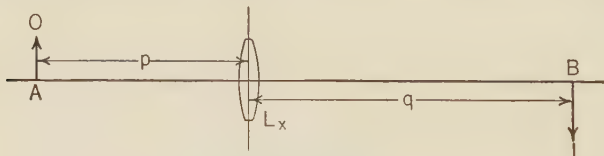
FIG. 25.

of the left principal focus of L_x. Measure the "object distance" p and the "image distance" q and solve for the focal length of L_x by use of the universal lens formula, which reads as follows:

$$\frac{1}{q} = \frac{1}{f} + \frac{1}{p}. \tag{12}$$

In the use of equation (12), the standard sign convention stated in Formula 3, Section 3, of the "Miscellaneous Introductory Notes" must be applied. For this case p in equation (12) is negative, and consequently we may write

$$\frac{1}{f} = \frac{1}{q} + \frac{1}{p} \tag{13}$$

or

$$f = \frac{pq}{p + q}. \tag{14}$$

38

In other words, for this case, merely measure p and q and substitute these numerical values in equation (14) and solve for f. f is equal to the product of p and q divided by the numerical sum of p and q.

Procedure:

Put OA at six different positions with respect to L_x. For each such position determine the corresponding p and q values in equation (14) and obtain an average value of f.

EXPERIMENT 11

Focal Length of a Converging Lens. Unit Magnification Method and $x_1 x_2 = f_1 f_2$ Method (Modification I)

(A)

Procedure:

1. Arrange the optical system as in Fig. 26. OA is an illuminated target, L_x is the test lens, and IB is the image of OA.
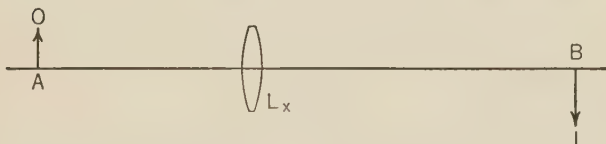
Fig. 26.

Arrange conditions for unit magnification, that is, adjust the distance between OA and its image IB so that OA and IB are of equal size. Neglecting the thickness of the lens, we may write for this condition as follows:

$$f = \frac{y}{4} \tag{15}$$

where f is the focal length of the lens and y is the distance between OA and IB.

2. Substitute the measured value of y in equation (15) and calculate the focal length of the lens supplied.

Question: Discuss the limitations to this method.

(B)

Procedure:

1. Arrange the optical system shown in Fig. 26 so that the distance between OA and IB is a minimum. Call this distance y. The focal length of the test lens L_x can be determined from the relation

$$f = \frac{y}{4} \tag{16}$$

where y represents the *minimum* distance between the target OA and its image IB.

Question: Discuss the limitations of this method and its relation to method (A) above.

(C)

Procedure:

1. Arrange the optical system as in Fig. 27.

2. By means of the collimator lens L_c and point source of illumination O put parallel light through the test lens L_x from the
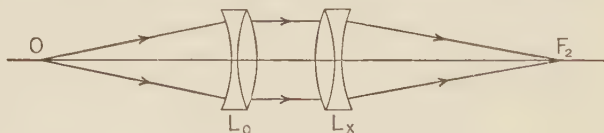
FIG. 27.

left, and by the use of a screen locate the focus F_2. (In order for parallel light to emerge from the right face of L_c, O must be at the left principal focus of L_c.)

3. Remove the collimator lens L_c, and without moving L_x determine the position of a point source of illumination O such that the distance between O and its image I is a minimum. For any case we may write

$$x_1 x_2 = f_1 f_2 \tag{17}$$

where x_1 is the distance between the object and the first principal focus and x_2 is the distance between the second principal focus and the image.

For this particular case (air on both sides of L_x)

$$f_1 = f_2 = f \tag{18}$$

where f is the focal length of lens L_x.

Also, for the case where the distance between O and I is a minimum

$$x_1 = x_2 = f_1 = f_2 = f \tag{19}$$

x_2 in this case is the distance between F_2 and I. The positions of points F_2 and I on the optical bench track have been determined. Therefore, measure the distance between F_2 and I. The result is the focal length sought.

Question: Compare method (C) with methods (A) and (B) above as regards the possible precision.

EXPERIMENT 12

Focal Length of a Converging Lens. $x_1x_2 = f_1f_2$ Method (Modification II)

Procedure:

1. Place the test lens L_x, Fig. 28, at some convenient position on the optical bench.
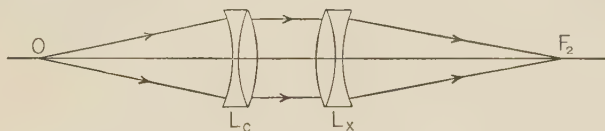
FIG. 28.

2. By means of collimator lens L_c and point source of illumination O put parallel light through L_x from the left, and by the use of a screen locate the focus F_2. (In order for parallel light to emerge from the right face of L_c, O must be at the left principal focus of L_c.)

3. By means of a collimator system consisting of L_c and O put parallel light through L_x from the right, and by the use of a screen
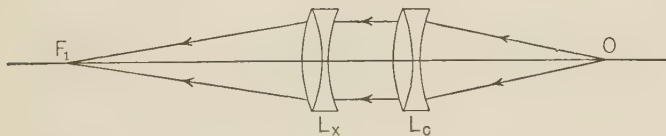
FIG. 29.

locate the focus F_1. (See Fig. 29.) L_x must occupy the same position as in step 2 above.

4. Positions of the foci F_1 and F_2 on optical bench track are now known.

5. Remove L_c, but do not move L_x.

6. Put O at any convenient position to the left of F_1 as shown in Fig. 30. By the use of a screen locate the image I. Let the
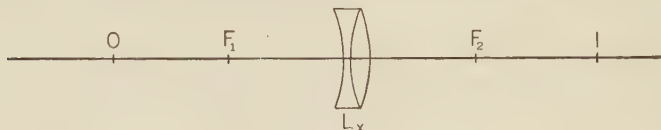
Fig. 30.

distance $OF_1 = x_1$ and the distance $IF_2 = x_2$. The following relation may now be written:

$$x_1x_2 = f_1f_2 \tag{20}$$

where f_1 and f_2 are the first and second focal lengths respectively. With air on both sides of the test lens

$$f_1 = f_2 = f \tag{21}$$

where f is the desired focal length of the lens.

Combining equations (20) and (21), we obtain

$$x_1x_2 = f^2 \tag{22}$$

Knowing the positions of O and F_1, and of I and F_2, we can measure x_1 and x_2, substitute these values in equation (22), and calculate f, the objective of this experiment.

7. Put O at six different positions. For each position determine the corresponding x_1 and x_2 values. Substitute each pair of values of x_1 and x_2 in equation (22) and obtain an average value of f.

EXPERIMENT 13

Focal Length of a Converging Lens. $x_1x_2 = f_1f_2$ Method (Modification III)

Procedure:

1. Using a point source of light O, Fig. 31, set the test lens for parallel light.

2. Without moving O, move test lens L_x a known distance x_1 to the right. (See Fig. 32.) The left focus F_1 of L_x has been

moved a distance x_1 to the right. F_2 is now the right focus of L_x. By the use of a screen locate the position of the image I, Fig. 32.
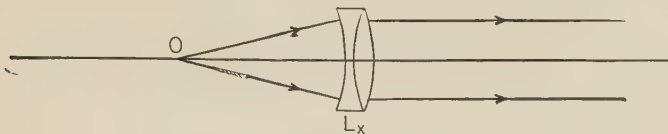
FIG. 31.
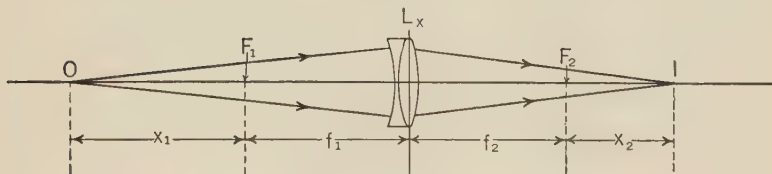
FIG. 32.

Let distance between O and I in Fig. 32 $= \Delta$. We may now write relations as follows:

$$x_1 + f_1 + f_2 + x_2 = \Delta \tag{23}$$

or

$$f_1 + f_2 + x_2 = \Delta - x_1 \tag{24}$$

or

$$x_2 = \Delta - x_1 - f_1 - f_2. \tag{25}$$

If the medium on both sides of the lens is air,

$$x_1x_2 = f^2 \tag{26}$$

where f is the focal length of the test lens L_x.

Combining equations (25) and (26),

$$x_1(\Delta - x_1 - 2f) = f^2 \tag{27}$$

Let $\quad\quad\quad\quad \Delta - x_1 = d.$ $\tag{28}$

Then

$$x_1(d - 2f) = f^2 \tag{29}$$

or

$$f^2 + 2fx_1 - x_1d = 0. \tag{30}$$

Measure the distances x_1 and d and substitute their values in equation (30). Solve equation (30) for f, the objective of this experiment.

Focal Length of a Converging Lens or Lens System. $y = f\theta$ Method (Modification I)

Method:

In Fig. 33, T_1T_2 is a target graduated to read directly in degrees when placed in the focal plane of the collimator lens L_c. In other words, the system consisting of the collimator lens L_c and the target T_1T_2 placed in the focal plane of L_c is the optical equivalent of a distant target graduated to read directly in degrees. L_x is the test lens. Parallel rays emerging from L_c are incident
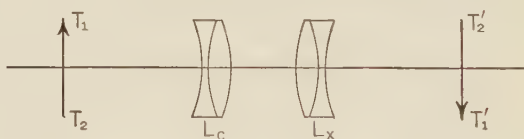
FIG. 33.

on L_x, and consequently a real image of T_1T_2 will be formed in the right-hand focal plane of L_x. $T'_1T'_2$ represents this image.

By any convenient means (dynameter, micrometer microscope, etc.) measure the length y of the image of a certain angular portion θ of T_1T_2. Note θ. The focal length of L_x can be calculated from the formula

$$f = \frac{y}{\theta}$$

where θ is expressed in radians and f is in the same length units as y.

Question: Show how the scale T_1T_2 is made to read directly in degrees when placed in the focal plane of L_c. In other words, show what determines the linear distance of T_1T_2 corresponding to a given angle.

Focal Length of a Converging Lens or Lens System. $y = f\theta$ Method (Modification II)

Procedure:

By means of any convenient device, such as a transit or sextant, measure the angle θ subtended at the observing station by two distant points A and B. Set up the test lens so that real images

A_1 and B_1 of A and B respectively are formed in the focal plane of the test lens. By means of a dynameter or micrometer microscope measure the distance y between A_1 and B_1 in the real image formed by the test lens.

Calculate the focal length of the lens or lens system from the relation

$$f = \frac{y}{\theta}$$

where θ is expressed in radians and f is in the same units as y.

EXPERIMENT 16

Focal Length of a Converging Lens or Lens System. $y = f\theta$ Method (Modification III)

Method:

This experiment involves the measurement of the focal length of a converging lens or lens system by methods involving the fundamental relation

$$y = f\theta$$

where y is the physical length of the image of a distant target;

θ is the angle in radians subtended by the distant target at the observing station;

f is the focal length of the lens system that forms the image of the distant target.

These methods will be listed below as (A), (B), (C), (D), (E), (F), and (G).

$$(A)$$

In Fig. 34, $T_1 T_2$ is a target placed in the focal plane of the collimator lens L_c. This target consists of a circular plate of thin
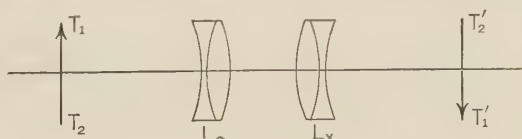
Fig. 34.

glass that has been silvered on one side. Five parallel straight lines have been ruled on this silvered surface, that is, five very narrow openings in the form of straight lines have been cut into

the silver.   Light can pass through these lines or openings, but
not through any other portion of the target.   The target has the
appearance shown in Fig. 35.   In addition to the five parallel

FIG. 35.

lines already mentioned there is a horizontal
diametral line ruled at right angles to the
other lines.
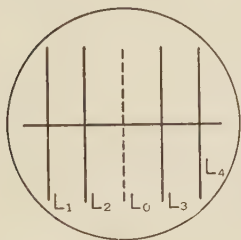
In Fig. 34, L_x is the test lens.   Parallel
rays emerging from L_c are incident on L_x,
and consequently a real image of T_1T_2 will
be formed in the right-hand focal plane of
L_x. $T'_1T'_2$ represents this image.

By any convenient means (dynameter,
micrometer microscope, etc.) measure the
distance y in the image between two of the parallel vertical lines
of the target.   Calculate the focal length of the test lens from
the equation

$$f = ky$$

where k is a constant which depends upon the two vertical lines
in the image between which the distance y has been measured.

If the lines selected are L_1 and L_4, $k = 10$.

If the lines selected are L_2 and L_3, $k = 20$.

If the lines selected are L_0 and either L_2 or L_3, $k = 40$.

Question: Make the necessary calculations for the target
employed in this experiment, assuming the focal length of the
collimator lens to be 315 mm.

(B)

Place the test lens L_x in the lens board of a camera and obtain
a photograph of the target T_1T_2.   When the photograph is com-
pleted proceed as in Method (A).

(C)

Proceed as in Method (B) except that the target and collimator
employed in Experiment 14 are to be substituted for the collimator
and target employed in Method (B) of this experiment.

(D)

Measure the diameter of the image of the sun formed by the
test lens.   Do the same for the moon.   In either case call the

diameter y. The corresponding value of θ can be obtained from the Nautical Almanac. Calculate f from the relation

$$f = \frac{y}{\theta}$$

(E)

Place the test lens in the lens board of a camera and obtain photographs of the sun and moon. When the photographs are completed measure y. θ can be obtained from the Nautical Almanac. Calculate f from the relation

$$f = \frac{y}{\theta}$$

(F)

In Fig. 36, $T_1 T_2$ is a target of any convenient form placed in the focal plane of the collimator lens L_c. The target employed
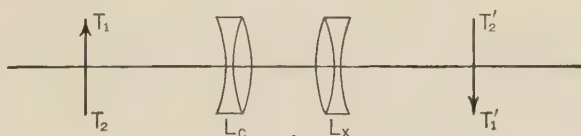
Fig. 36.

in part (A) of this experiment or the one employed in Experiment 14 will be suitable. Place the test lens L_x in position and measure the distance y_x in the image corresponding to any given portion of the target $T_1 T_2$. Now replace the test lens L_x by a standard lens of known focal length f_s. Measure the distance y_s in the image corresponding to the *same* portion of the target used when the test lens was in position. Calculate the focal length f_x of the test lens from the relation

$$f_x = f_s \frac{y_x}{y_s} \tag{31}$$

(G)

Proceed as in (F), except that an actual distant target is to be substituted for the collimator target.

EXPERIMENT 17

Focal Length of a Converging Lens System.   Nodal-Point Method
(Reflection Auto-Collimation Modification)

Method:

In Fig. 37, F is a point source of light, L_x is the test lens, MM' is a plane mirror placed normal to the principal axis.   If the distance FE is equal to the focal length of the lens, object and image will coincide at F.   By inclining MM' slightly, an image can be
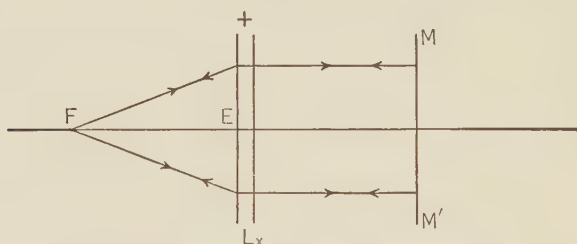
Fig. 37.

found near F.   If the test lens is rotated slightly about an axis passing through E and approximately perpendicular to the principal axis, no motion of the image will result.   In order to determine the focal length of the test lens, it is necessary to measure the distance FE when two conditions are satisfied.   These two conditions are:

(a) A sharply defined image of F must be formed near F itself.

(b) No motion of this image must occur when the test lens is rotated slightly about the axis of the nodal-point carriage.

The positions of F and E are to be read from the graduated track of an optical bench. E is the principal point of emergence of L_x.

Results Required:

Focal lengths of converging lens systems supplied.

<div align="center">EXPERIMENT 18</div>

Focal Length of a Converging Lens or Lens System.   Nodal-Point Method, Using Precision Optical Bench with Collimator Target or with Distant Target

<div align="center">(a) USING COLLIMATOR TARGET</div>

Method:

Arrange the optical system as shown in Fig. 38. $T_1 T_2$ is any convenient form of target located in the focal plane of the collima-
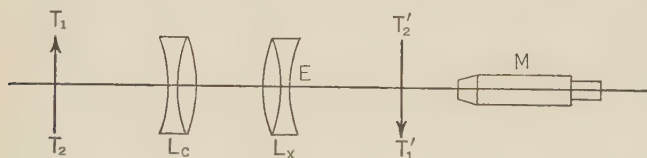
FIG. 38.

tor lens L_c.   Emergent parallel rays from L_c are incident on the test lens L_x and form a real image $T'_1 T'_2$ in the right-hand focal plane of L_x.   Focus the microscope M on this image.   If the test lens L_x is rotated about a vertical axis passing through the emergent principal point E, no motion of the image as seen through the microscope will occur.   The optical system must be arranged so that two conditions are satisfied as follows:

1. The image of the target as seen through the microscope is sharply defined, that is, it is in focus.

2. No motion of this image occurs when the test lens is rotated slightly about the axis of the nodal-point carriage.

The focal length of the test lens is the distance between the image referred to above and the axis of rotation mentioned in condition 2.   Two methods for determining this distance will be described.   These methods will be called I and II in what follows.

I. With the microscope at a fixed position move the carriage supporting the test lens to a position such that conditions 1 and 2 above are satisfied.   Read this position of the nodal-point carriage on the optical bench track.   Now remove the test lens and replace it by a diaphragm having a very small hole at its center. Move the nodal-point carriage toward the microscope to a position where a sharply defined image of the small hole in the diaphragm is observed on looking into the microscope.   By means

of the adjusting devices on the nodal-point carriage set the position of the small hole so that its image viewed through the microscope does not move when the nodal-point slide is rotated about the main axis of its carriage. Read this position of the nodal-point carriage on the optical bench track. The distance between this position of the nodal-point carriage and the first position read above is the focal length of the test lens L_x.

II. With the microscope at a fixed position move the nodal-point carriage supporting the test lens to a position such that conditions 1 and 2 above are satisfied. Read this position of the nodal-point carriage on the optical bench track. Remove the test lens and replace it by a lens of known focal length. Now move the nodal-point carriage supporting the lens of known focal length to a position such that conditions 1 and 2 above are satisfied. Read this position of the nodal-point carriage on the optical bench track. The distance between this position of the nodal-point carriage and the corresponding position when the test lens is used is the difference between the focal lengths of the two lenses. Knowing this difference and the focal length of the standard lens, the focal length of the test lens can be readily calculated.

(b) Using Distant Target

An actual distant target may be used instead of the collimator target. In this case remove the collimator target and arrange the optical system so that light from the distant target is incident on the test lens. Follow exactly the same procedure throughout as that employed in (a).

EXPERIMENT 19

Focal Length of a Converging Lens or Lens System. Special Method No. I

Method:

This method depends upon a fundamental relation which reads as follows:

$$E.F. = \frac{B.F. + F.F. + \sum \frac{d}{n}}{2} \tag{32}$$

where $E.F.$, $B.F.$, and $F.F.$ are the effective focal length, "back

focus," and "front focus" respectively of the converging lens or lens system and $\sum \frac{d}{n} = \frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3} + \ldots$ is the "apparent thickness" of the system.

Procedure:

1. Arrange the optical system as in Fig. 39. L_c is a collimator lens having an illuminated target $T_1 T_2$ in its left focal plane. Par-
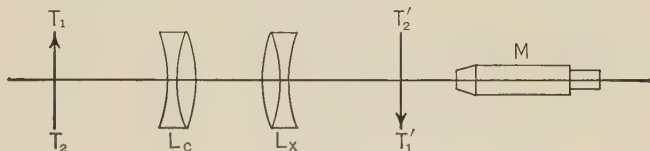
FIG. 39.

allel light emerges from L_c and is incident on the test lens L_x. An image $T'_1 T'_2$ of $T_1 T_2$ is formed in the right-hand focal plane of L_x and the microscope M is focused on this image. Read the position of L_x on the optical bench track. Now move L_x toward the microscope to the position where the image of a small cross scratched on the lens surface at the position of the vertex or pole on the right face of lens L_x is seen sharply focused on looking into the microscope. Read this position of L_x on the optical bench track. The distance between the first and second positions of L_x is the back focus, that is, $B.F.$ in equation (32).

2. Turn lens L_x around so that the parallel light emerging from L_x is now incident on the opposite face of L_x and proceed as in step 1. The distance between the first and second positions of L_x on the optical bench track is the "front focus," that is, $F.F.$ in equation (32).

3. Two methods will be employed for determining the apparent thickness $\sum \frac{d}{n}$. The methods will be designated as (a) and (b).
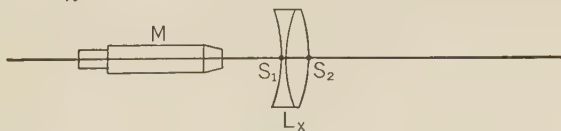
FIG. 40.

(a) Focus the microscope M, Fig. 40, first on the small cross scratched on the lens surface to indicate the position of pole S_1

and then on the small cross that indicates the position of pole S_2. The distance between the two positions of the microscope is $\sum \dfrac{d}{n}$. It is assumed that the lens is not moved. If it is more convenient to keep the microscope stationary and move the lens, then of course the distance between the two positions of the lens is $\sum \dfrac{d}{n}$.

(b) Arrange the optical system as in Fig. 41. x is a point source of illumination placed approximately at the left focus of collimator lens L_c. Approximately parallel light emerges from
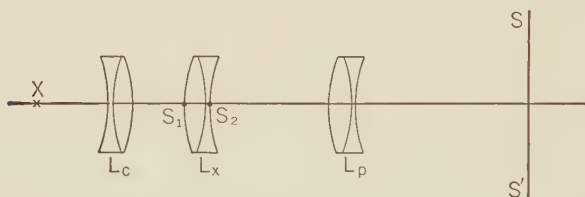
FIG. 41.

L_c. This light is used merely to illuminate the small crosses at the poles S_1 and S_2 of the test lens L_x. L_p is a "projection" lens, the object of which is to form a magnified image of S_1 or S_2 on the screen SS'. For a fixed position of L_p and SS' move L_x along the optical bench track to a position such that a magnified image of S_2 is formed on the screen SS'. Read this track position of L_x. Now move L_x to a position such that an image of S_1 is formed on the screen SS'. The distance between the two track positions of L_x is equal to the desired quantity $\sum \dfrac{d}{n}$, i.e., it is the "apparent distance" between S_1 and S_2. This distance might be called an "optical distance."

4. B.F. was determined in step 1. F.F. was determined in step 2. $\sum \dfrac{d}{n}$ was determined in step 3. Substitute these values in equation (32) and solve for E.F., which is being sought.

Question: Discuss the merits of this method as a precision method.

Determination of Focal Length of a Converging Lens or Lens System by the Hartmann Method

Theory:

In Fig. 42, E_1 and E_2 are the principal points of a converging lens L_x having a principal focus at F_2. DD is an opaque diaphragm placed perpendicular to the principal axis. O and O' are very small
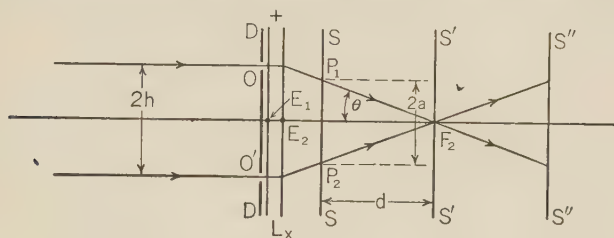
FIG. 42.

circular openings in DD equidistant from the principal axis of the test lens L_x and they are separated by a distance $2h$. It is evident from the figure that the focal length f can be expressed by the relation:

$$f = h \cot \theta \tag{33}$$

This relation is general and will apply to any lens or lens system. Assume a screen placed at some position SS to the right of the lens, and to the left of the principal focus F_2. Two luminous spots P_1 and P_2 will appear on SS. Represent the distance between these spots by $2a$. Now assume the screen to be moved slowly to the right. When the screen has been moved a distance d so that it is exactly in the focal plane corresponding to the principal focus F_2 the two luminous spots will coalesce. This is the $S'S'$ position of the screen in Fig. 42.

It can be easily shown that

$$\tan \theta = \frac{a}{d} \tag{34}$$

Substituting equation (34) in equation (33),

$$f = \frac{hd}{a} \tag{35}$$

In many cases it is more convenient to make observations with the screen at a position $S''S''$ to the right of F_2 instead of at position SS to the left of F_2. This is particularly true for short focal lengths. Equation (35) applies in either case.

The method in this experiment consists in measuring h, a, and d and substituting these values in equation (35).

Procedure:

The procedure should be obvious from the "Theory." Make several check determinations.

Note: This method can be carried out photographically.

<div align="center">EXPERIMENT 21</div>

Focal Lengths and Cardinal Points of Ramsden's Eye-Piece

Theory:

Ramsden's eye-piece consists of two plano-convex lenses (convex surfaces toward each other) of equal focal lengths, separated by a distance d equal to two-thirds the focal length of either. The latter distance is measured between the principal point of one lens component and the adjacent principal point of the second lens component.
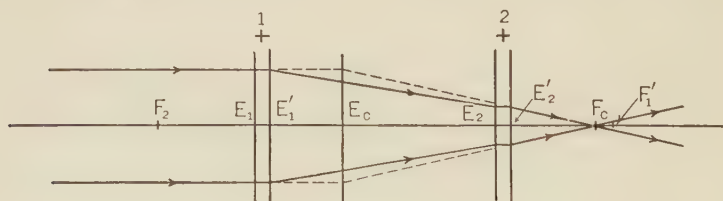
FIG. 43.

For light traveling from left to right, Fig. 43 shows the principal point of emergence E_c of the combination and the corresponding principal focus F_c. The distance E_cF_c in Fig. 43 is the effective focal length of the combination.

E_1 and E'_1 are the principal points of lens component 1. F'_1 is the principal focus of lens component 1 corresponding to principal point E'_1. The focal length of lens component 1 is the distance $E'_1F'_1$.

E_2 and E'_2 are the principal points of lens component 2. F_2 is the principal focus of lens component 2 corresponding to

principal point E_2.   The focal length of lens component 2 is the distance E_2F_2.

The left principal focus F_1 of lens component 1 and the right principal focus F'_2 of lens component 2 are not shown in Fig. 43 because of lack of space. F_1 is as far to the left of E_1 as F'_1 is to the right of E'_1. F'_2 is as far to the right of E'_2 as F_2 is to the left of E_2.   If f_1 and f_2 are the focal lengths of the component lenses separated by distance d, then

$$f_c = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{36}$$

where f_c is the effective focal length of the combination.

For this lens combination $f_1 = f_2 = f$ and $d = 2f/3$.   Making these substitutions in equation (36), we get

$$f_c = \frac{3f}{4} \tag{37}$$

Equation (37) shows that the effective focal length f_c of the combination is equal to three-fourths the focal length of either lens component.

The "back focus" $[B.F.]_c$ of the combination can be expressed in general as follows:

$$[B.F.]_c = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \tag{38}$$

In this case $f_1 = f_2 = f$, and $d = 2f/3$.   Making these substitutions, equation (38) becomes

$$[B.F.]_c = \frac{f}{4} \tag{39}$$

Equation (39) shows that the "back focus" of the combination is equal to one-fourth the focal length of either lens component.

For light traveling from right to left Fig. 44 shows a principal point E'_c and a corresponding principal focus F'_c for the combination.

Referring now to Figs. 43 and 44 it is important for the reader to notice the following:

$$E'_1 E_c = E_2 E'_c$$

$$E'_2 F_c = E_1 F'_c$$

$$E_c F_c = E'_c F'_c$$

In other words, the four cardinal points of the combination and the eight cardinal points of the two component lenses are symmetrically spaced with respect to each other. The combination is positive or converging turned either way toward the incident light. The Ramsden's eye-piece may be called a symmetrical lens combination.
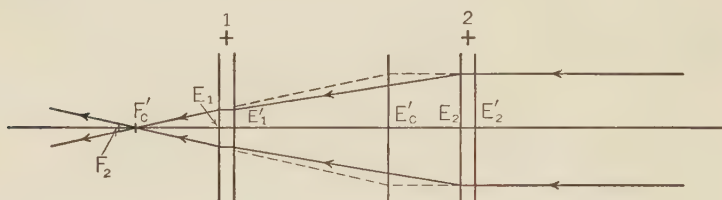
FIG. 44.

Procedure:

Determine by use of nodal-point carriage and optical bench the relative positions of the cardinal points and the values of the effective focal length and "back focus" of the Ramsden's eye-piece supplied.

Results Required:

Diagram of optical system for each measurement. Neat drawing for each case, with dimensions, showing relative positions of the four cardinal points of combination and the eight cardinal points of the component lenses.

EXPERIMENT 22

Focal Lengths and Cardinal Points of Huyghens' Eye-Piece

Theory:

Huyghens' eye-piece consists of two identical plano-convex lenses (both convex sides facing the incident light); the focal lengths of the two lenses are in ratio of 3 to 1 or 2 to 1, the distance between the lenses is one-half the sum of the focal lengths, and the lens of longer focal length faces the incident light.

Figure 45 represents a Huyghens' eye-piece. The ratio of the focal length of lens component 1 to the focal length of lens component 2 is 2 to 1.

For light traveling from left to right and the lens component

of longer focal length toward the incident light, Fig. 45 shows the principal point of emergence E_c of the combination and the corresponding principal focus F_c. The distance $E_c F_c$ in Fig. 45 is the effective focal length of the combination. E_1 and E'_1 are the principal points of lens component 1. F'_1 is the principal focus of lens component 1 corresponding to principal point E'_1. The focal length of lens component 1 is the distance $E'_1 F'_1$. E_2 and E'_2 are the principal points of lens component 2. F_2 is the prin-
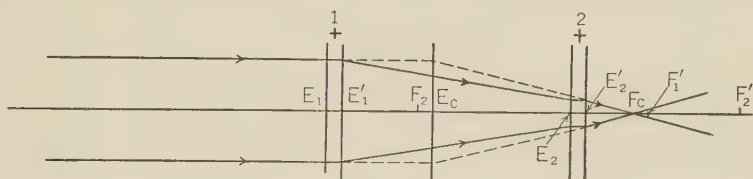
Fig. 45.

cipal focus of lens component 2 corresponding to principal point E_2. The focal length of lens component 2 is the distance $E_2 F_2$.

The left principal focus of lens component 1 is not shown in Fig. 45 because of lack of space. F_1 is as far to the left of E_1 as F'_1 is to the right of E'_1.

If f_1 and f_2 are the focal lengths of the component lenses separated by distance d, then

$$f_c = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{40}$$

where f_c is the effective focal length of the combination. For this combination, $f_1 = 2f_2$ and $d = 3f_2/2$. Making these substitutions in equation (40), we get

$$f_c = \tfrac{4}{3} f_2 \tag{41}$$

Equation (41) shows that the effective focal length of the combination is equal to four-thirds the focal length of the lens component of shorter focal length.

The "back focus" $[B.F.]_c$ of the combination can be expressed in general as follows:

$$[B.F.]_c = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \tag{42}$$

In this case $f_1 = 2f_2$ and $d = 3f_2/2$. Making these substitutions, equation (42) becomes

$$[B.F.]_c = \frac{f_2}{3} \tag{43}$$

Equation (43) shows that the "back focus" of the combination is positive and is equal to one-third the focal length of the lens component of shorter focal length. The distance E'_2F_c in Fig. 45 is the "back focus."

For light traveling from right to left, Fig. 46 shows a principal point E'_c and a corresponding *virtual* principal focus F'_c for the combination. The distance $E'_cF'_c$ is the effective focal length of the combination.

In Fig. 46, the lens of shorter focal length faces the incident
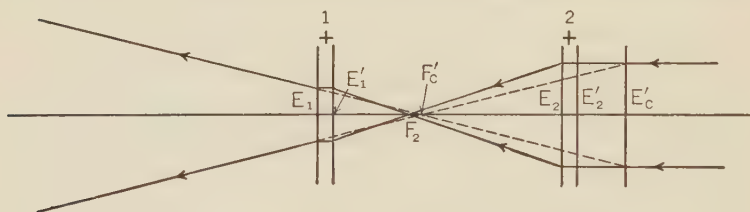
FIG. 46.

light. It is important to notice that the combination used this way is negative or diverging. The "back focus" in this case is

$$[B.F.]_c = -\frac{2f_2}{3} \tag{44}$$

Equation (44) shows that the "back focus" of the combination used this way is negative and is equal to two-thirds the focal length of the lens component of shorter focal length. The distance $E_1F'_c$ in Fig. 45 is the "back focus."

Recapitulation at this point indicates that the Huyghens' eye-piece is a converging or positive lens system if the lens component of longer focal length faces the incident light; if the lens component of shorter focal length faces the incident light the system is diverging or negative. In order to be of any value as an ocular in a telescope the Huyghens' eye-piece must be arranged with the lens component of shorter focal length nearer the observer's eye. The lens component of longer focal length is called the

"field lens"; the one of shorter focal length is called the "eye lens." The Huyghens' eye-piece is sometimes called a "negative" eye-piece because when it is used in a telescope the image formed by the objective must fall to the rear of the field lens.

Procedure:

Determine by use of nodal-point carriage and optical bench the relative positions of the cardinal points and the values of the effective focal length and "back focus" of the Huyghens' eye-piece supplied

Results Required:

Diagram of optical system for each measurement. Neat drawing for each case, with dimensions, showing relative positions of the four cardinal points of the combination and the eight cardinal points of the component lenses.

EXPERIMENT 23

Focal Lengths and Cardinal Points of Telephoto Lens Combination

Theory:

The telephoto lens combination consists of a simple positive or converging lens and a simple negative or diverging lens "air separated" by such a distance as to obtain a comparatively long effective focal length.

For light incident on the converging lens component, Fig. 47 shows the principal point of emergence E_c and the corresponding
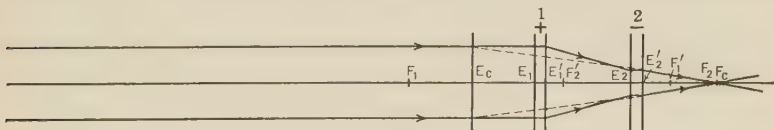
FIG. 47.

principal focus F_c. The distance E_cF_c in Fig. 47 is the effective focal length of the combination.

E_1 and E'_1 are the principal points of lens component 1. F'_1 is the principal focus of lens component 1 corresponding to principal point E'_1.

F_1 is the principal focus of lens component 1 corresponding to principal point E_1. The focal length of lens component 1 is the distance E_1F_1 or the distance $E'_1F'_1$.

E_2 and E'_2 are the principal points of lens component 2. F'_2 is the *virtual* principal focus corresponding to principal point E'_2 and F_2 is the *virtual* principal focus corresponding to principal point E_2.   The focal length of lens component 2 is the distance E_2F_2 or the distance $E'_2F'_2$.

If f_1 and f_2 are the focal lengths of the component lenses separated by distance d, then

$$f_c = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{45}$$

where f_c is the effective focal length of the combination.

The purpose in the telephoto lens combination is to have an effective focal length which is large compared to the physical length of the combination itself.   With such an arrangement it is possible to form reasonably large images of distant inaccessible objects.   The size and weight of the optical system in this case will be small compared to the size and weight of a system of equal effective focal lengths employing a simple lens.   To illustrate what has just been said, consider a telephoto arrangement having constants as follows:

$$f_1 = + \ 200 \text{ mm.}$$
$$f_2 = - \ \ 50 \text{ mm.}$$
$$d = \ \ \ \ 160 \text{ mm.}$$

Substituting the above values in equation (45) we get

$$f_c = + \ 1000 \text{ mm.}$$

The "back focus" $[B.F.]_c$ of the combination can be expressed in general as follows:

$$[B.F.]_c = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \tag{46}$$

Substituting the data above in equation (46) we get

$$[B.F.]_c = + \ 200 \text{ mm.}$$

Using the data assumed and the corresponding results obtained, and applying them to Fig. 47, we get

$$E'_1F'_1 = \ \ \ \ 200 \text{ mm.}$$
$$E'_2F'_2 = \ \ - 50 \text{ mm.}$$

$$E'_1E_2 = \quad 160 \text{ mm.}$$
$$E_cF_c = \quad 1000 \text{ mm. (effective focal length)}$$
$$E'_2F_c = \quad 200 \text{ mm. ("back focus")}$$
$$E_1F_c = \quad 360 \text{ mm. (approximately)}$$

The significant fact shown by these figures is that an effective focal length of 1000 mm. is obtained by the use of an optical arrangement which has a physical length of approximately 360 mm. This result is due, as shown by Fig. 47, to the fact that the principal point E_c is located in space well to the front of the material parts of the combination.

For light incident on the diverging component of the telephoto lens combination, Fig. 48 shows the principal point E'_c and the
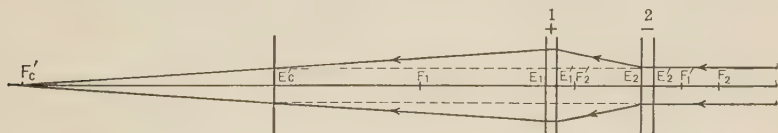
FIG. 48.

corresponding principal focus F'_c of the combination.   Employing equation (46) in this case to calculate the "back focus" we get

$$[B.F.]_c = + 4200 \text{ mm.}$$

Using the data assumed and the corresponding results obtained, and applying them to Fig. 48, we get

$$E'_1F'_1 = \quad 200 \text{ mm.}$$
$$E'_2F'_2 = \quad - 50 \text{ mm.}$$
$$E'_1E_2 = \quad 160 \text{ mm.}$$
$$E'_cF'_c = \quad 1000 \text{ mm.}$$
$$E_1F'_c = \quad 4200 \text{ mm.}$$
$$E'_2F'_c = \quad 4360 \text{ mm. (approximately)}$$

The physical length of the optical system in this case is approximately 4360 mm.   Consequently, it is impractical to use this lens combination with the diverging component toward the incident light.

Procedure:

Determine by use of nodal-point carriage and optical bench the relative positions of the cardinal points and the values of the effective focal length and "back focus" of the telephoto lens combination supplied.

Results Required:

Diagram of optical system for each measurement. Neat drawing for each case, with dimensions, showing relative positions of the four cardinal points of the combination and the eight cardinal points of the component lenses.

Note: Figs. 47 and 48 are drawn to scale but the constants are not in the same proportion as the constants in the example given above.

SECTION III

FOCAL LENGTHS OF DIVERGING LENSES AND LENS SYSTEMS

EXPERIMENT 24

Determination of Focal Length of a Simple Diverging Lens of Short Focal Length

Theory:

In Fig. 49, L_1 is the diverging test lens and L_2 is an auxiliary converging lens. The effective focal length $E.F.$ of the lens com-
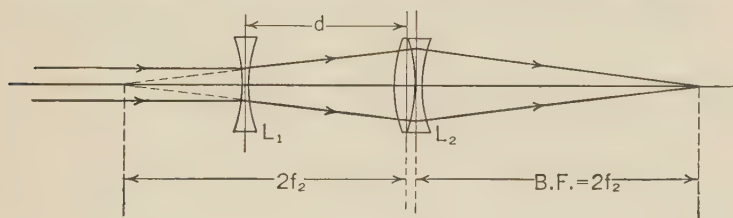
FIG. 49.

bination (lenses L_1 and L_2) can be expressed as follows:

$$E.F. = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{47}$$

where f_1 is the focal length of diverging test lens L_1;

f_2 is the focal length of auxiliary converging lens L_2;

d is the distance between adjacent principal points of lenses L_1 and L_2.

The "back focus" $B.F.$ of the lens combination can be expressed as follows:

$$B.F. = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \tag{48}$$

The meanings of symbols f_1, f_2, and d are the same as in equation (47).

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In equation (48) let $B.F.$ have the particular value equal to $2f_2$, i.e., the "back focus" of the combination is positive and is equal to twice the focal length of the auxiliary converging lens. If $B.F. = 2f_2$, then equation (48) may be written

$$2f_2 = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} \tag{49}$$

or

$$d = f_1 + 2f_2 \tag{50}$$

Substituting equation (50) in equation (47)

$$E.F. = -f_1 \tag{51}$$

Equation (51) suggests a method for measuring the focal length f_1 of the diverging lens L_1.   The method is as follows:

1. Select an auxiliary converging lens of known focal length f_2.

2. Arrange a lens combination as shown in Fig. 49, consisting of the diverging test lens L_2 and the auxiliary converging lens mentioned in step 1.

3. Adjust the distance d so that the measured $B.F.$ of the lens combination is equal to $2f_2$.

4. When condition 3 is satisfied, measure the $E.F.$ of the lens combination.

5. The measured $E.F.$ referred to in step 4, is, according to equation (51), numerically equal to the focal length f_1 of the diverging test lens.

Procedure:

Obvious from above "Theory."

Results Required:

To be specified at time of experiment.

EXPERIMENT 25

Determination of Focal Length of a Simple Diverging Lens. Galilean Telescope Method

Theory:

(a) REFLECTION AUTO-COLLIMATION

In Fig. 50, S is a point source of light placed at the focus of the collimator lens L_c, L_1 is an auxiliary converging lens, L_2 is the diverging test lens, and MM' is a plane mirror. If the rays

emerging from L_2 are parallel to each other and to the principal axis, and if the mirror MM' is normal to these rays, they will be reflected back over their original path, and as a result object and image will coincide at S.

When the rays emerging from L_2 are parallel to each other and to the principal axis, lenses L_1 and L_2 form a lens system having a focal length equal to infinity. The system is therefore telescopic.
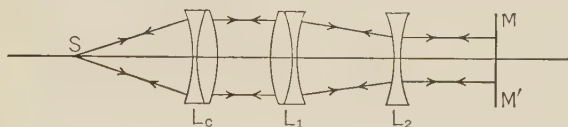
Fig. 50.

A telescopic system consisting of a converging objective and a diverging ocular is known as the Galilean telescope.

The effective focal length $E.F.$ of the lens system consisting of lenses L_1 and L_2 is

$$E.F. = \frac{f_1 f_2}{f_1 + f_2 - d} \quad (52)$$

where f_1 is the focal length of lens L_1;

f_2 is the focal length of lens L_2;

d is the distance between the adjacent principal points of L_1 and L_2.

If $E.F. = \infty$, then

$$f_1 + f_2 - d = 0 \quad (53)$$

or

$$f_2 = d - f_1 \quad (54)$$

The method in this case therefore consists in arranging the optical system as in Fig. 50 and adjusting the relative positions of the auxiliary converging lens L_1 of known focal length f_1 and L_2 so that object and image coincide at S. When this condition has been established, measure the distance d. The focal length f_2 of the diverging test lens L_2 can now be determined by use of equation (54).

(b) DISTANT TARGET

Consider the optical system of Fig. 51. L_1 is an auxiliary converging lens of known focal length f_1, and L_2 is the simple

diverging test lens. Parallel rays from a distant target are
incident on L_1. If the distance d between L_1 and L_2 is adjusted
so that the system is telescopic, i.e., so that an observer who
places his eye back of L_2 will see an image of a distant target, then

$$E.F. = \frac{f_1 f_2}{f_1 + f_2 - d} = \infty \tag{55}$$

or

$$f_2 = d - f_1 \tag{56}$$

The method in this case consists in arranging the optical sys-
tem as in Fig. 51 and adjusting the relative positions of L_1 and

FIG. 51.

L_2 so that a well-defined image of the distant target is observed
when the observer's eye is placed to the rear of L_2. When this
condition has been established measure d. The focal length of the
diverging test lens can now be determined by use of equation (56).

Procedure:

The lens called L_1 in method (a) is to be the lens employed as
L_1 in method (b); the lens called L_2 in method (a) is to be employed
as L_2 in method (b).

Results Required:

To be specified at time of experiment.

EXPERIMENT 26

Measurement of Focal Length of a Diverging Lens or Lens System By the " Telephoto Lens Combination " Method

Method:

Arrange the optical system as in Fig. 52.

$T_1 T_2$ is a target placed in the focal plane of the collimating
lens L_c. L_1 is an auxiliary converging lens of known focal length f_1.
L_2 is the diverging lens being tested. L_1 and L_2 are arranged so
as to form a converging system (telephoto combination), a real

image $T'_1T'_2$ of T_1T_2 being formed to the right of L_2.   Measure:
(a) the size I_x of the real image formed by this system, and
(b) the distance d between the adjacent principal points of L_1
and L_2.

Remove L_2 and measure the size I_s of the image of T_1T_2
formed by L_1.
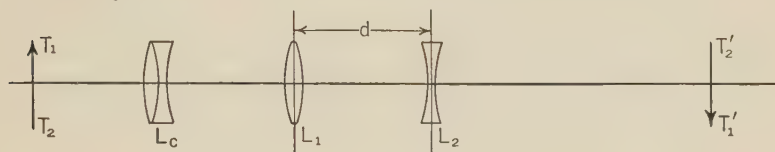
FIG. 52.

Calculate the focal length f_c of the system from the formula

$$f_c = f_1 \frac{I_x}{I_s} \tag{57}$$

where f_1 is the known focal length of L_1.

Calculate the focal length f_2 of the diverging lens from the
formula

$$f_c = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{58}$$

Procedure:

Obvious from "Method" discussed above.

Results Required:

Six different determinations of f_2, using a different value of d
in each case.

EXPERIMENT 27

Determination of Focal Length of a Simple Diverging Lens of Short Focal Length, Using the " One to One " Micrometer Microscope

Theory:

The principle involved in this method is the same as that in
Experiment 24.   For the sake of convenience, use is made in this
case of a "one to one" micrometer microscope.   The necessary
optical arrangement in this case is shown in Fig. 53. L_1 is the
simple diverging test lens, L_2 and E are, respectively, the objective
lens and the ocular of the "one to one" micrometer microscope.

MS is a direct-reading linear scale graduated to tenths of milli-meters. This so-called "mil scale" is placed so that two condi-tions hold as follows:

(a) The distance between MS and L_2 is equal to $2f_2$ where f_2 is the focal length of L_2.

(b) MS is in the left focal plane of ocular E.

When conditions (a) and (b) have once been established, L_2, MS, and E are fixed in position with respect to each other.

It is proved in the "Theory" of Experiment 24 that the focal length of the diverging test lens is numerically equal to the $E.F.$
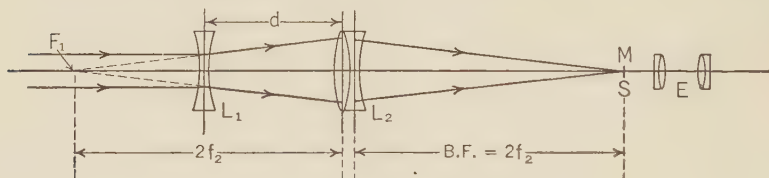
Fig. 53.

of the lens combination when the $B.F.$ of the combination is equal to $2f_2$.

Assume, now, that parallel light from a distant target is incident on lens L_1 in Fig. 53. Keeping in mind that the com-ponent parts L_2, MS, and E of the "one to one" micrometer microscope cannot be moved with respect to each other, adjust the distance d so that a sharply defined image of the distant target appears in the field of view of the micrometer microscope. Call the measured length of this image y. The $E.F.$ of the lens combination consisting of L_1 and L_2 is

$$E.F. = y \cot \theta \qquad (59)$$

where θ is the angle subtended by the distant target at the observ-ing station. But we know from equation (51) of Experiment 24 that

$$E.F. = -f_1 \qquad (60)$$

Therefore, having measured $E.F.$, we now have determined the numerical value of f_1.

Procedure:

Obvious from above "Theory."

Results Required:

To be specified at time of experiment.

<div align="center">EXPERIMENT 28</div>

Focal Length of a Diverging Lens.   Nodal-Point Method I

Theory:

In Fig. 54, S is a point source of illumination placed outside the left principal focus F_L of an auxiliary converging lens L_c. Rays emerging from L_c are converging toward the point F_x. The diverging test lens L_x is placed in such a position as to intercept the rays from L_c and make them parallel to the principal
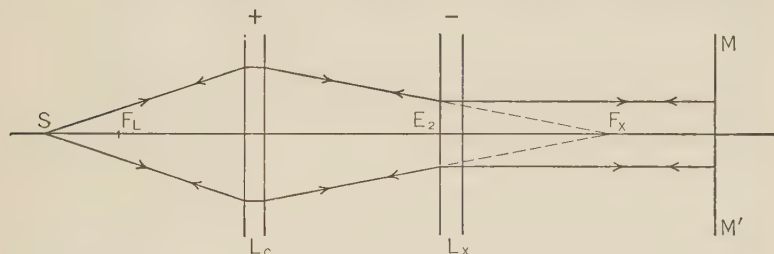
<div align="center">FIG. 54.</div>

axis on emerging from L_x. MM' is a plane mirror placed normal to the rays emerging from L_x.   The rays incident on MM' will retrace their original path.   As a consequence, object S and its image will coincide.   By inclining MM' very slightly, S and its image may be seen side by side.   If lens L_x is rotated about an approximately vertical axis passing through E_2 no motion of the image of S will occur.   The position of E_2 can be determined by means of the nodal-point carriage and graduated track.   The focal length of L_x is the distance E_2F_x in Fig. 54.   The position of F_x can be determined as follows:

Keep S and L_c in the same positions as in locating the position of E_2.   Remove L_x and locate point F_x by means of a screen which can be moved along the graduated track.   The distance on the graduated track between the position of the axis of rotation of the nodal-point carriage which produces no motion of the image and the position of the screen mentioned above is the focal length of the diverging test lens.

Procedure:

Obvious from above "Theory."

Results Required:

Focal lengths of diverging lens supplied.

EXPERIMENT 29

Focal Length of a Diverging Lens or Lens System. Nodal-Point Method II

Theory:

In Fig. 55, S is a point source of light at the principal focus of a collimator lens L_c. L_x is the diverging test lens, L_A is an auxiliary converging lens, and SS' is an opaque white screen. Assume this optical system so arranged that an image I of S is formed on

FIG. 55.

SS'. Rotation of L_x about a vertical axis through its emergent principal point E_2 (not shown in Fig. 55) will produce no motion of image I. The position of this principal point E_2 can be read from the graduated track of the optical bench. Now assume the point source S to be placed at I (L_A must not be disturbed) and the test lens L_x to be removed. A real image of S formed by L_A will now be found at a point F, which is the principal focus of the test lens L_x. The position of F on the optical bench track can be found by means of a white opaque screen. The distance between E_2 and F is the focal length of the diverging lens or lens system L_x.
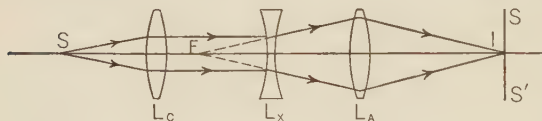
Procedure:

Obvious from above "Theory."

Results Required:

Focal lengths of lenses supplied.

EXPERIMENT 30

Determination of Focal Length of a Diverging Lens or Lens System by an Extension of the Principle of the Hartmann Method

Theory:

In Fig. 56, E_1 and E_2 represent the principal points of a simple diverging lens having a principal focus at F_2. DD is an opaque

diaphragm placed perpendicular to the principal axis, having very small circular openings at O and O'. O and O' are equidistant from the principal axis and are separated by a distance $2h$. It is evident from the figure that the focal length f can be expressed by the relation

$$f = h \cot \theta \tag{61}$$

This relation is general and will apply to any lens or lens system. Assume a screen placed at some position SS to the right of the lens.
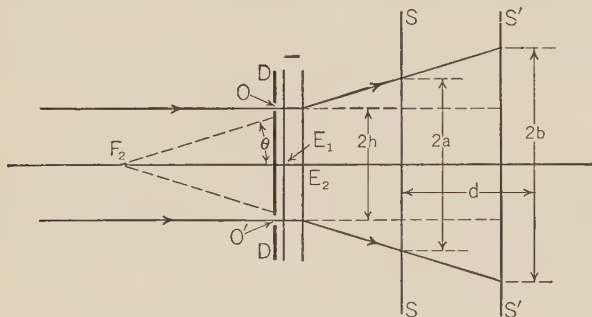
FIG. 56.

Two luminous spots will appear on SS. Represent the distance between these spots by $2a$. Now assume the screen to be moved by a distance d to a position $S'S'$. Represent the distance between the luminous spots in this case by $2b$. It can easily be shown that

$$\cot \theta = \frac{d}{b - a} \tag{62}$$

Substituting equation (62) in equation (61)

$$f = \frac{hd}{b - a} \tag{63}$$

This method consists, therefore, in measuring h, d, b, and a, and substituting these values in equation (63).

Procedure:

The procedure should be obvious from the "Theory." Make several check determinations.

SECTION IV

LENS ABERRATIONS

EXPERIMENT 31

Determination of Longitudinal Spherical Aberration of a Converging Lens or Lens System by the Hartmann Method

Theory:

For any simple spherical converging lens of appreciable aperture, edge rays incident parallel to the principal axis do not pass through the principal focus. This is illustrated in Fig. 57. The
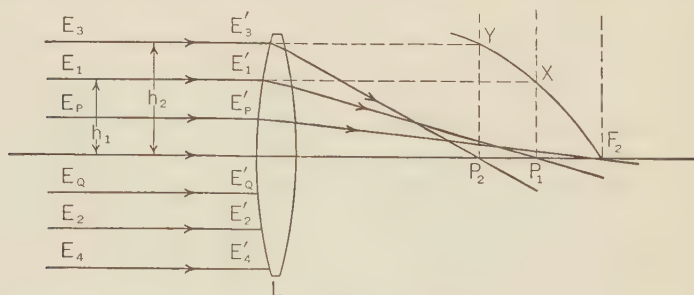
FIG. 57.

paraxial rays $E_P E'_P$ and $E_Q E'_Q$ incident parallel to the principal axis converge to the principal focus F_2. Edge rays $E_1 E'_1$ and $E_2 E'_2$ converge to point P_1; edge rays $E_3 E'_3$ and $E_4 E'_4$ converge to P_2. The distances h_1 and h_2 are called "incidence heights." For zone $E'_1 E'_2$ the longitudinal spherical aberration is $P_1 F_2$; for zone $E'_3 E'_4$ the longitudinal spherical aberration is $P_2 F_2$. If perpendiculars cutting the incident rays produced in X and Y be erected at P_1 and P_2, we have at hand a graphic means of representing the variation of longitudinal spherical aberration with incidence height. The points F_2, X, and Y are on a curve which coordinates incidence height on the y axis and spherical

aberration on the x axis.   Since spherical aberration for any zone is measured with respect to F_2, the spherical aberration for the paraxial zone is zero and the curve passes through O. O (point F_2 in Fig. 57).

The general scheme of the Hartmann method for measuring longitudinal spherical aberration may be explained by reference to Fig. 58. DD is an opaque diaphragm placed perpendicular to the principal axis, having very small circular openings at O, O', Q, Q', R, R'. O and O' are equidistant from the principal axis, as are also Q and Q' and R and R'.   The incidence heights corresponding to openings Q and Q' and to R and R' are the distances
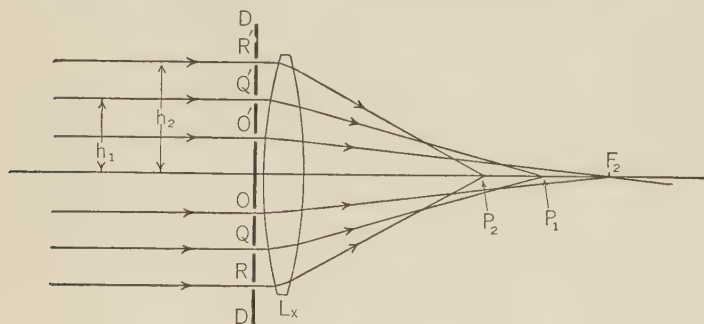
Fig. 58.

h_1 and h_2, respectively, in Fig. 58.   Assume a beam of rays incident from the left and parallel to the principal axis. L_x is the test lens.   Consider O and O' sufficiently near the principal axis so that the rays passing through O and O' may be considered paraxial rays.   These rays will converge to a point F_2, which is the principal focus of L_x.   If L_x is a simple spherical lens, the edge rays which pass through Q and Q' will converge to a point P_1 and the edge rays which pass through R and R' will converge to a point P_2.

Procedure:

By means of a screen which can be moved along the graduated track of an optical bench locate the points F_2, P_1, P_2, and corresponding points for other openings in the diaphragm DD.   Use monochromatic light and adjust so that the principal axis of the test lens is parallel to the optical bench track.   Repeat with the lens reversed.   Determine the effective focal length by a method to be specified at time of experiment.

The incidence heights corresponding to each pair of openings in the diaphragm DD will be found marked on the diaphragm.

Results Required:

Spherical-aberration curve similar to the curve shown in Fig. 57 for each lens supplied for both directions of incidence plotted: (a) from test results, and (b) from calculated results. Conclusions regarding the longitudinal spherical aberration of: (a) simple spherical lenses, (b) carefully designed lens systems.

EXPERIMENT 32

Determination of Axial Chromatism (Chromatic Aberration) of a Converging Lens or Lens System

Procedure:

Arrange the optical system as in Fig. 59. S is a point source of monochromatic illumination placed at the focus of a well-
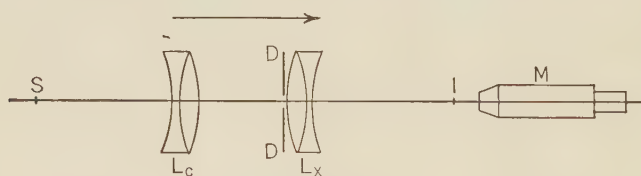
FIG. 59.

corrected collimator lens L_c. L_x is the test lens and M is a microscope which can be moved along a graduated track in a direction parallel to the principal axis of the test lens. DD is a diaphragm with a small circular opening at its center by means of which the test lens can be "stopped down" to the paraxial region so as to eliminate longitudinal spherical aberration. I is the monochromatic image of S. Provision must be made so that the wave length or color of S can be varied. The particular way in which this is to be done is immaterial and depends upon the facilities available. For each one of several wave lengths determine the position of microscope M when a sharply defined image I of S is seen in the field of view of M.

Repeat the above observations for two edge zones of the test lens.

Determine the focal length of the test lens for some one of the wave lengths already employed.

Note: The collimator arrangement of Fig. 59 can be replaced by a "distant" monochromatic point source of illumination. In many cases a distance of 50 to 100 feet is sufficient.

Results Required:

Curve coordinating wave length in Ångströms on the x axis and focal length in millimeters on the y axis. Use scales such that the points on the curve can be read with the degree of accuracy obtained in the experiment.

Tabulated results showing variation of position of image I with wave length and with lens zone and incidence height.

EXPERIMENT 33

Determination of Curvature of Field and Astigmatism of a Converging Lens System

Method:

Arrange the optical system as in Fig. 60. x is an illuminated target in the form of either a very small pinhole or a small cross
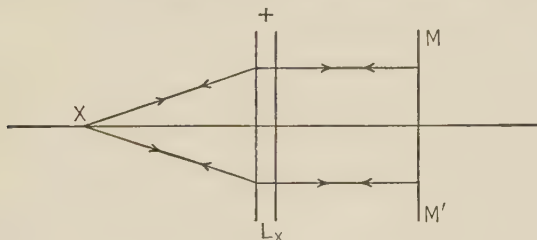
FIG. 60.

with one narrow horizontal line and one narrow vertical line. L_x is the test lens mounted on a nodal-point carriage which can be moved along the graduated track of an optical bench. MM' is a plane mirror.

Adjust the optical system so that three conditions are satisfied in the order listed below:

1. Target and its image are seen side by side, i.e., the illuminated target x is in the left focal plane of L_x.

2. Axis of rotation of nodal-point carriage passes through the principal point of emergence of test lens L_x, i.e., image of x does not move on rotating test lens L_x through a small angle about a vertical axis.

3. The principal axis of test lens L_x is parallel to the graduated track of the optical bench.

When the three conditions just mentioned are satisfied read the position of the nodal-point carriage on the graduated track of the optical bench. Read also the position of x. The difference between these two readings is the effective focal length of test lens L_x.

Now rotate the test lens L_x 5 degrees clockwise about the vertical axis of the nodal-point carriage. Move the nodal-point carriage toward x until the secondary (horizontal) focal line appears near x. Read the track position of the nodal-point carriage.

Move the nodal-point carriage toward x until the primary (vertical) focal line appears near x. Read the track position of the nodal-point carriage. Repeat the readings for the secondary and primary focal lines for a 5-degree rotation from the original position of L_x in the counterclockwise direction.

Repeat the procedure of the previous paragraphs for angles of 10, 15, 20, 25, 30, 35, and 40 degrees.

The method to be employed in reducing the data obtained will be illustrated by a numerical example. In Fig. 61 the distance $XE_2 = 759.9$ mm. represents the effective focal length of the test lens; that is, the target is at zero on the track and the nodal-point carriage is at 759.9 mm. when the three conditions mentioned above are satisfied. For an angular rotation of 25 degrees of the test lens about the vertical axis of the nodal-point carriage, the track reading of the nodal-point carriage is 715 mm. when the secondary (horizontal) focal line appears near x and 590 mm. when the primary (vertical) focal line appears near x.

The distance XY in Fig. 61 is equal to $759.9/\cos 25 = 838.47$ mm. The focal plane which contains the line E_2Y will be called the "flat image plane" or "ideal image plane." For a perfectly flat field and no astigmatism at 25 degrees, primary and secondary focal lines would degenerate to a point at Y and the track reading would be 838.47 mm. Since the actual track reading observed for the secondary focal line is 715 mm., this focal line falls short of the "flat image plane" by the distance $SY = 123.47$ mm. measured obliquely. The primary focal line falls short by the oblique distance $PY = 838.47 - 590 = 248.47$ mm. Measured parallel to the principal axis of the lens, the secondary focal line falls short of the flat image plane 111.9 mm., and the primary focal

line falls short 225.19 mm.   The distances SA = 111.9 mm. and PB = 225.19 mm. are the values of the curvature of field for the secondary and primary focal lines respectively for oblique rays making an angle of 25 degrees with the principal axis.

The distance between the secondary and the primary focal lines (715 − 590 = 125 mm.) is known as the "Astigmatic Difference."

Figure 61 shows the departure from a "flat field" for several different oblique incident rays.   The angles which these different
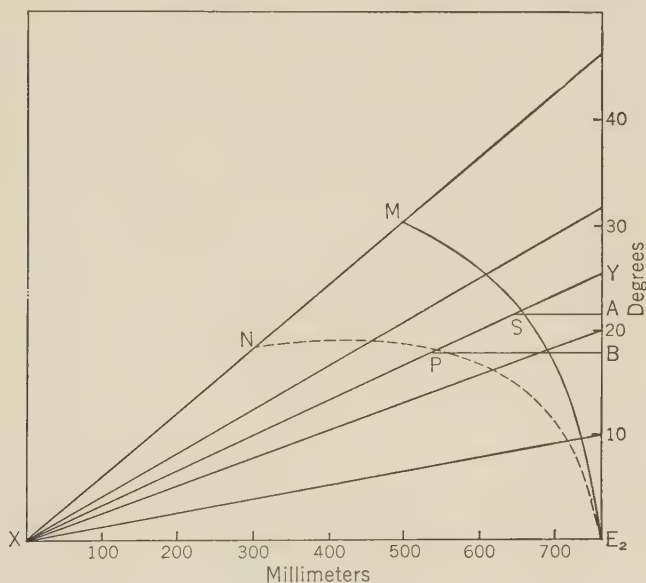
FIG. 61.

rays make with the principal axis are the ordinates in Fig. 61. The unbroken curve E_2SM is the locus of the secondary focal lines, and the broken curve E_2PN is the locus of the primary focal lines. These two curves illustrate two important points for oblique rays as follows:

1. The image of a point is not a point.   The nearest approximation to a point image is the "circle of least confusion," which is located between the secondary and primary focal lines.

2. The field is not "flat" but has a general shape probably intermediate between the curves E_2SM and E_2PN in Fig. 61.

The curvature of field for the secondary and primary focal lines corresponding to any oblique ray can be expressed as per cents as follows:

Divide the amount by which the secondary and primary focal lines respectively fall short of the focal plane in a direction parallel to the principal axis by the effective focal length. For the data assumed above for an oblique ray incident at an angle of 25 degrees with the principal axis, the curvature of field for the secondary focal line is 14.73 per cent and for the primary focal line, 29.63 per cent.

The reflection auto-collimation arrangement of the optical system shown in Fig. 60 is suggested because of its convenience. A substitute arrangement is shown in Fig. 62. x is the illuminated
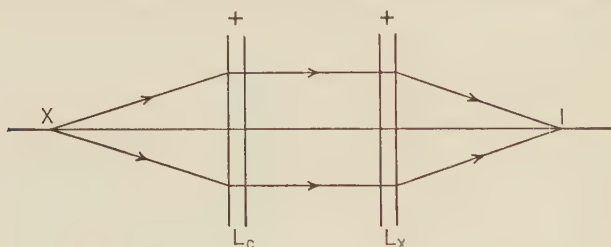
Fig. 62.

target placed in the left focal plane of collimator lens L_c. Parallel light is incident on the test lens L_x and is brought to a focus at I. Another arrangement consists in using a distant illuminated target. The procedure in the latter two arrangements is, in general, the same as that outlined above in detail for the reflection auto-collimation method.

In each of the three arrangements already mentioned the test lens is rotated about a vertical axis in order that the object point may be displaced a certain angular amount from the principal axis. If it is not convenient to rotate the test lens, the same result can be obtained by the use of a fairly distant target consisting of about five luminous points arranged along a straight line perpendicular to the principal axis of the test lens. One of the luminous points must be on the axis and each of the others is displaced from the axis by a definite angular amount.

If circumstances permit, the student is advised to employ this latter arrangement to check results already obtained.

Results Required:

Curvature of field for secondary and primary focal lines as per cent of effective focal length, and astigmatic differences for oblique rays incident at angles of 5, 10, 15, 20, 25, 30, 35, and 40 degrees with the principal axis. Curves similar to those in Fig. 61 to show the departure from a flat field.

Comment on probable performance of this lens as a wide-angle anastigmat.

EXPERIMENT 34

Measurement of Distortion of a Converging Lens System

Method:

$$(A)$$

Arrange the optical system as in Fig. 63. T_1T_2 is an illuminated target having the appearance shown in Fig. 64. This
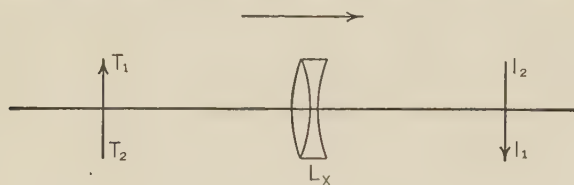
Fɪɢ. 63.

target consists of two sets of straight lines ruled at right angles to each other so as to form a series of small squares that are all exactly alike. I_1I_2 is a screen consisting of a white surface ruled with black lines. The ruled surface on I_1I_2 is an exact duplicate of the ruling on T_1T_2. Adjust the optical system for unit magnification at the center of the field, that is, arrange the relative positions of T_1T_2, the test lens L_x, and the screen I_1I_2 so that the image of the central square of T_1T_2 formed on I_1I_2 is exactly the same size as the central square of I_1I_2. T_1T_2 and
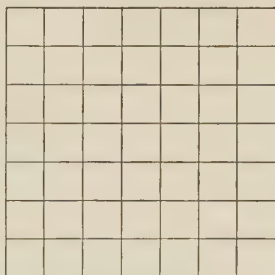
Fɪɢ. 64.

I_1I_2 must be perpendicular to the optical bench track. Also adjust so that the image of the central small square of T_1T_2 coincides exactly with the central small square ruled on I_1I_2.

The principal axis of the test lens must be perpendicular to T_1T_2 and to I_1I_2 and parallel to the optical bench track.

Examine the screen I_1I_2 and note whether all portions of the image of T_1T_2 coincide exactly with the corresponding rulings on I_1I_2. If perfect coincidence occurs on all parts of I_1I_2, the test lens is free from distortion. If distortion is present note whether it is of the barrel or the pin-cushion type. Repeat the above procedure for any magnifications other than unity for which the lens may be intended to be used. In such cases the screen I_1I_2 must be ruled for the particular magnification selected.

(B)

Replace the target T_1T_2 of Fig. 64 by an illuminated linear horizontal scale which is perpendicular to the optical bench track. Replace I_1I_2 by any convenient linear scale which is horizontal, and perpendicular to the optical bench track. Adjust for unit magnification at the center of the field and measure the magnification at several other parts of the field. The principal axis of the test lens must be perpendicular to both scales and parallel to the optical bench track. Now adjust both scales so that they are perpendicular to the optical bench track and vertical and proceed as before.

Repeat the above procedure for any magnifications other than unity for which the lens may be intended to be used.

Results Required:

Tabulated results and conclusions.

EXPERIMENT 35

Foucault or " Knife-Edge " Test of Lens
for Detecting Aberrations

Theory:

This method depends upon the fact that in a well-corrected lens system emergent rays will intersect in approximately a point (a focus), and as a result the form of light pattern received on either side of the focus is an evenly illuminated disc. If a knife-edge is moved slowly across the line of sight exactly in the focal plane of a well-corrected lens system, the illumination will be cut

off abruptly as the knife-edge crosses the axis, for an observer whose eye is on the axis and at a short distance to the rear of the focus.   If the lens system is not well corrected spherically or chromatically, there is no position of the knife-edge at which the illumination disappears abruptly.   Either the inner or the outer portion of the light pattern appears unequally illuminated.   If white light is employed, the disc may appear differently colored for different positions of the knife-edge along the axis; in such case there are color errors present in the lens system.

Procedure:

Test the objective lenses supplied by the method outlined in the above "Theory" and report results.

SECTION V

INTERFERENCE

EXPERIMENT 36

Determination from Newton's Rings of:

(a) Wave Length of Light

(b) Radius of Curvature of a Lens

Theory:

In Fig. 65, x is a point source of monochromatic light placed at the focus of collimator lens L_c, PP is a thin glass plate, and L_x is a double convex lens resting on a plate of glass GG. M is a micrometer microscope used to observe and measure the Newton's rings formed by the thin air film in the neighborhood of the point of contact C between the lens L_x and plate of glass GG. R is the radius of curvature of the side of lens L_x in contact with glass plate GG.
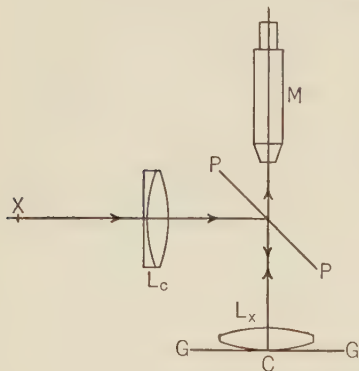
The wave length λ of the light employed, the radius of curvature R of the surface of the lens L_x in contact with glass plate GG, and the diameter D_n of the nth bright ring are connected by the formula

$$\lambda = \frac{D_n^2}{2R(2n + 1)} \tag{64}$$

Procedure:

1. Using a lens of known R, determine the wave length of the light supplied. Measure D_n by means of the micrometer microscope.

2. Using monochromatic light of known wave length, determine the radius of curvature of the lens supplied.

EXPERIMENT 37

Measurement of Wave Length of Light by Means of the Plane-Transmission-Diffraction-Grating Spectrometer

Procedure:

Arrange the optical system as in Fig. 66. S and L_c are the slit and collimator lens, respectively, of the spectrometer, GG' is the plane transmission-diffraction grating, L_o and L_E are the objective and ocular, respectively, of the spectrometer telescope.
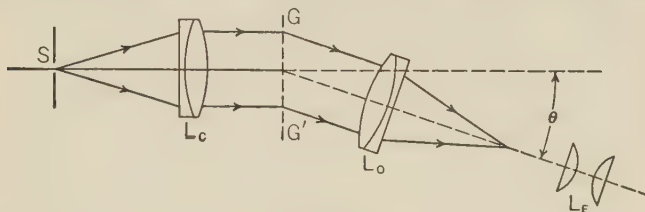
FIG. 66.

Determine the angle θ for the first-order spectrum to the right and left for each wave length to be measured and calculate the wave length from the following equation:

$$\lambda = d \sin \theta \qquad (65)$$

where d is the distance between the centers of two adjacent grating spaces.

Results Required:

Wave lengths specified.

Question: What is the difference between a grating spectrum and a prismatic spectrum?

EXPERIMENT 38

Michelson Interferometer Measurements:

(a) Determination of Wave Length

(b) Calibration of a Linear Scale

Theory:

The optical system of the Michelson interferometer is shown in Fig. 67 (a). S is a point source of monochromatic light placed at

the focus of collimator lens L_c, A and B are plane parallel glass plates, C and D are plane glass surfaces heavily silvered on the sides toward B and A respectively, i.e., C and D are plane mirrors. Mirror D can be displaced in a direction parallel to that of the light incident upon it.

Assume a ray of monochromatic light from S incident at point O of the left or "front" face of plate A. Part of this incident light is reflected at O. (We will not concern ourselves with the light thus reflected.) The remainder of the incident light, except a negligible amount absorbed by plate A, passes through glass plate A to a point O' on the rear surface of A. At O' part of the light
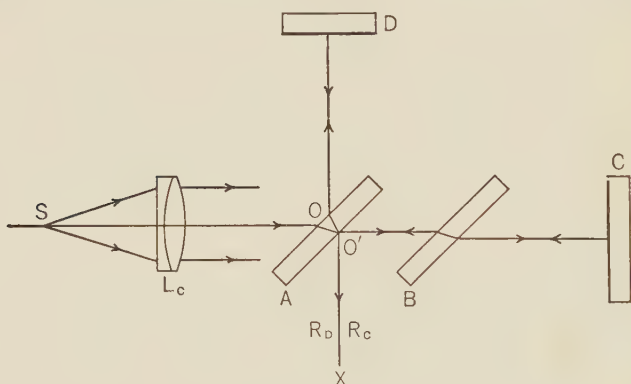
Fig. 67 (a).

emerges from A, passes through the glass plate B, and is reflected by the plane mirror C back to the rear surface of A, where it is in part reflected as ray R_c. Part of the incident beam which arrived at O' will be reflected at O' and will reach the plane mirror D. At D this light will be reflected. Part of this light will emerge from A as ray R_D. Light finally arrives at some point X. Some of this light traverses one of the paths described above and the rest of it traverses the other path. When the instrument is properly adjusted, the rays R_c and R_D coincide. Plate B is introduced to make the *glass optical paths* for both parts of the original ray equal in length; the ray R_D traverses the plate A three times, and the ray R_c traverses the same thickness of glass when the compensator B is introduced.

If the path difference between R_c and R_D is an odd number of

half wave lengths, the two systems of waves will interfere *destructively* at X and the result at this point will be darkness. The path difference over the two "routes," including mirrors C and D respectively, can be altered by moving the carriage supporting D along ways by means of an accurately made screw.

As D is moved along the ways a particular point on the rear surface of A will appear alternately bright and dark. A movement of mirror D through a physical distance of $\lambda/2$ (change in optical path equal to λ) will cause the displacement of any bright band into the position previously occupied by the adjacent bright band.

Some light is reflected from the front surface of A, but its effect may be rendered negligible by "half-silvering" the rear surface of A. A "half-silvered" mirror is one having a silver deposit of such thickness that about half of the incident light is transmitted and about half is reflected. The rear surface of A is usually half-silvered.

Procedure:

Adjust mirror D so that its distance from the rear surface of A is approximately the same as that of C from A. This adjustment is suggested because it is easier to find the fringes when the distance between the mirror D and the virtual image of mirror C is small. This distance will be called the "distance between the mirrors."

Place the source of monochromatic light at S approximately at the principal focus of the collimator lens L_c of short focal length. Place a diaphragm containing a pinhole aperture between A and L_c with the pinhole approximately on the principal axis of L_c. On looking toward mirror D from X three images of the pinhole should appear. One image is formed by reflection at the front surfaces of A and D; the second is formed by reflection at the rear surface of A and the front surface of D; the third is formed by reflection from the front surface of C and the rear surface of A. Interference fringes in monochromatic light are found by bringing this third image into coincidence with either of the other two by means of the adjusting screws upon which the mirror C rests. If, however, it is desired to find the fringes in white light, the second and third of these images should be brought into coincidence, because then the two paths of the light in the instrument are symmetrical, i.e.,

each is made up of a given distance in air and a given thickness of glass. When the paths are symmetrical, the fringes are always approximately arcs of circles as described above. If, however, the first and third images are made to coincide, then the two optical paths are unsymmetrical, i.e., the path from A to C has more glass in it than that from A to D, and in this case the fringes may be ellipses or equilateral hyperbolae, because of the astigmatism which is introduced by the two plates A and B. It is quite probable that the fringes will not appear when the two images of the pinhole seem to have been brought into coincidence, simply because the eye cannot judge with sufficient accuracy for this purpose when the two are really superposed. To find the fringes then, it is only necessary to move the adjusting screws slightly back and forth. As the instrument has been here described, the second image lies to the right of the first.

When the fringes have been found the student should practice adjustment until he can produce at will the various forms of fringes. Thus the circles appear when the distance between the mirrors is not zero, and when the mirror D is strictly parallel to the virtual image of C. The accuracy of this adjustment may be tested by moving the eye sideways and up and down while looking at the circles. If the adjustment is correct, any given circle will not change its diameter as the eye is thus moved. To be sure, the circles appear to move across the plates because their center is at the foot of the perpendicular dropped from the eye to the mirror D, but their apparent diameters are independent of the lateral motion of the eye. For this reason it is advisable to use the circular fringes whenever possible.

To find the fringes in white light, adjust so that the monochromatic fringes are arcs of circles. Move the carriage rapidly by intervals of a quarter turn or so of the wheel. When the region of the white-light fringes has been passed, the curvature of the fringes will have changed sign, i.e., if the fringes were convex toward the right, they will now be convex toward the left. Having thus located within rather narrow limits the position of the mirror D, which corresponds to zero difference of path, it is only necessary to replace the sodium light by a source of white light, and move the mirror D by means of the worm slowly through this region until the fringes appear.

These white-light fringes are strongly colored with the colors

of Newton's rings. The central fringe, the one that indicates exactly the position of zero difference of path, is, as with Newton's rings, black. This black fringe will be entirely free from color, i.e., perfectly achromatic, if the plates A and B are of the same piece of glass, are equally thick, and are strictly parallel. If they are matched plates, i.e., if they are made of the same piece of glass and have the same thickness, their parallelism should be adjusted until the central fringe of the system is perfectly achromatic. When this is correctly done the colors of the bands on either side of the central one will be symmetrically arranged with respect to the central black fringe.
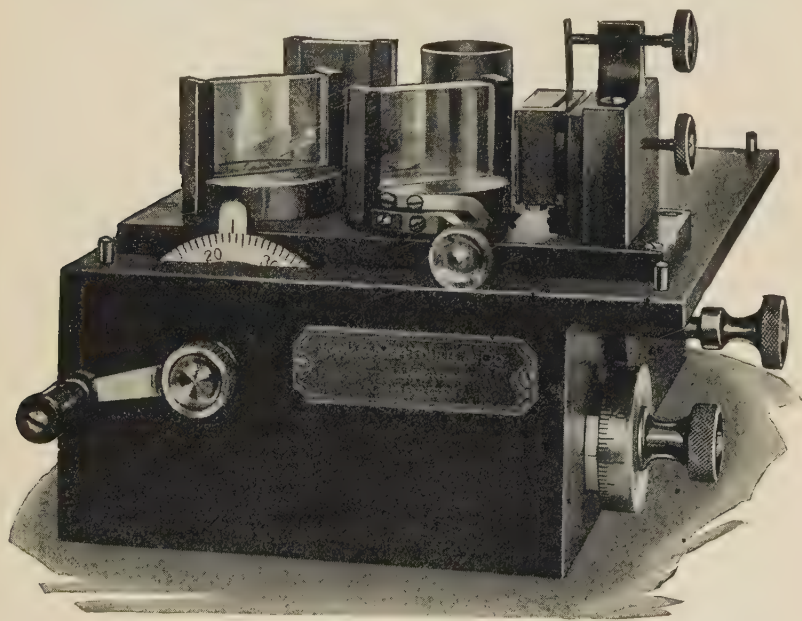
FIG. 67 (*b*).

Figure 67 (*b*) shows a modern laboratory Michelson interferometer.

Measurements to be Made:

  (*a*) Wave length of light from monochromatic source.
  (*b*) Calibration of scale.

Acknowledgment:

  The procedure for this experiment and some of the exact language are taken from Mann's "Manual of Advanced Optics."

EXPERIMENT 39

Resolving-Power Measurements

(a) DETERMINATION OF THE LIMIT OF RESOLUTION
OF AN APERTURE OF WIDTH a

Procedure:

Illuminate the grating target with monochromatic light. Place the slit of variable width over the objective of the telescope and adjust the width a to about 2 mm. Now determine the maximum distance between telescope and grating target for which resolution occurs.

Repeat this procedure for other values of slit width a.

It is shown in treatises on physical optics that

$$\theta = \frac{\lambda}{a} = \frac{d}{D} \qquad (66)$$

where θ is the limiting angle of resolution. For practical purposes this angle will be referred to as the resolving power;

λ is the wave length of the light employed;

a is the slit width;

d is the width of a grating-target space;

D is the distance between telescope objective and grating target when resolution ceases.

Use your observed data to test equation (66).

(b) RESOLVING POWER OF THE HUMAN EYE

Procedure:

Measure the resolving power of your eyes.

(c) RESOLVING POWER OF LENSES AND TELESCOPES

Theory:

It is customary to consider that the resolving power of a lens or telescope can be expressed by the relation:

$$R = \frac{127}{d} \qquad (67)$$

where R is the limiting angle of resolution or the resolving power in seconds, and d is the diameter of the effective aperture expressed in millimeters.

Procedure:

Test equation (67), using the lenses and telescopes supplied. Use several values of d in each case.

Note: It has been shown that equation (67) can be very misleading for a practical telescope. There is very good reason to believe that the practical resolving power of a telescope depends not only upon d as shown by equation (67), but also upon the diameter of the exit pupil and the effective focal length of the objective lens. This means, of course, that there is some connection in a practical telescope between resolving power and magnification.

Additional Procedure:

Obtain sufficient data to establish an empirical relation connecting resolving power of a practical telescope with such factors as magnification, diameter of exit pupil, and focal length of objective lens.

PHOTOMETRY

EXPERIMENT 40

Fundamental Photometric Measurements on Tungsten and Carbon-Filament Lamps

(a) Luminous Intensity

Theory:

The principle of the method for determining the luminous intensity of a lamp may be understood by reference to Fig. 68. S_1 is a standard lamp of known intensity I_1, S_2 is the test lamp of unknown intensity I_2, and P is a screen or "photometer head" normal to and movable along the line S_1 S_2. The Lummer–
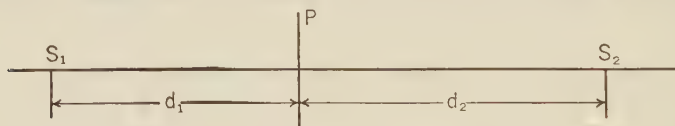
Fig. 68.

Brodhun type of photometer head is very satisfactory. The student should consult some standard text-book for the principle of the Lummer–Brodhun or other type of photometer head. The left side of P is illuminated by S_1 and the right side by S_2, and both sides may be viewed simultaneously by the observer. When equality of illumination is established on the two sides of P, we may write:

$$E_1 = E_2 \qquad (68)$$

where E_1 and E_2 are the illuminations from S_1 and S_2 respectively. But

$$E_1 = \frac{I_1}{d_1{}^2} \qquad (69)$$

and

$$E_2 = \frac{I_2}{d_2{}^2} \qquad (70)$$

Combining equations (68), (69), and (70),

$$\frac{I_1}{d_1{}^2} = \frac{I_2}{d_2{}^2} \tag{71}$$

Equation (71) forms the basis of the method employed in this experiment. I_1 is known, d_1 and d_2 can be read on a graduated track, and therefore I_2 can be calculated.

(b) Luminous Efficiency in Watts per Candle

Different types of incandescent lamps vary in luminous efficiency, that is, in the power input necessary to give equivalent intensity. By use of ammeter and voltmeter in the circuit of each of the lamps mounted on the photometer bench, the power input to each lamp may be measured. Knowing the candle-power of each lamp the efficiencies in watts per candle may be determined.

Procedure for (a) and (b):

The Reichsanstalt type of photometer bench and a modified form of the Lummer–Brodhun type of photometer head will be employed in this experiment. Determine the luminous intensity of each lamp supplied. By means of ammeter and voltmeter or by wattmeter measure the power input in each case.

(c) Relation between Intensity and Voltage

The intensity of an electric incandescent lamp varies with the voltage applied as follows:

$$\frac{I_1}{I_2} = \left(\frac{E_1}{E_2}\right)^c \tag{72}$$

where I_1 and I_2 are the intensities at the voltages E_1 and E_2 respectively and c is a constant for the type of filament. Equation (72) holds for a range of about 10 per cent above and below normal lamp voltage. For tungsten $c = 3.6$ and for "treated" carbon $c = 5.55$.

(d) Relation between Luminous Efficiency and Voltage

For an electric incandescent lamp the luminous efficiency in watts per candle varies with the voltage applied as follows:

$$\frac{L_1}{L_2} = \left(\frac{E_1}{E_2}\right)^k \tag{73}$$

where L_1 and L_2 are the luminous efficiencies at the voltages E_1 and E_2 respectively, and k is a constant for the type of filament. Equation (73) holds for a range of about 10 per cent above and below normal voltage. For tungsten $k = -2.03$ approximately and for "treated" carbon $k = -3.48$ approximately.

Procedure for (c) and (d):

Measure the luminous intensity and watts per candle at voltages to be specified at time of experiment.

Results Required:

Luminous intensity and luminous efficiency of each lamp at normal voltage. Values of c and k in equations (72) and (73) respectively for each lamp supplied.

Conclusions:

Discuss fully the significance of the results.

EXPERIMENT 41

Direct-Reading Photometer Bench

Method:

In Fig. 69, S_1 and S_2 are lamps of luminous intensities I_1 and I_2 respectively, and P is the photometer head. When equality of

Fig. 69.

illumination is established on the two sides of P we may write

$$\frac{I_1}{d_1{}^2} = \frac{I_2}{d_2{}^2} \tag{74}$$

or

$$I_2 = \left(\frac{I_1}{d_1{}^2}\right)d_2{}^2 \tag{75}$$

From equation (75) it is evident that if I_1 and d_1 are kept constant, I_2 varies directly as $d_2{}^2$, or

$$I_2 = kd_2{}^2 \tag{76}$$

where

$$k = \left(\frac{I_1}{d_1{}^2}\right)$$

This suggests a direct-reading scheme to be employed in connection with the ordinary photometer bench. Assume a bar S_1S_2, Fig. 69, 3000 mm. long, which is to indicate at its left end 90 candle-power, the zero of the bar being at the right end.

If a 90-candle-power lamp is placed at the right end of the bar the illumination I_1 at the left end is

$$I_1 = \frac{90}{(3)^2} = 10 \text{ meter candles}$$

If a 22.5-candle-power lamp is placed at the right end the illumination I'_1 at the middle of the bar (1500 mm. from the source) is

$$I'_1 = \frac{22.5}{(1.5)^2} = 10 \text{ meter candles}$$

Proceeding in this manner it is possible to calculate the position on the photometer bar where the illumination will be 10 meter candles for any candle-power between 0 and 90 located at the right end of the bar. Assume the photometer bar to be graduated in this manner. For example, at the center of the bar the new scale will be marked 22.5; 1730 mm. from the right it will be marked 30; 3000 mm. from the right end it will be marked 90; etc.

To make a measurement, place a lamp of known candle-power at S_2 and place the photometer-head carriage at the position on the photometer bar where the scale reading is equal to the candle-power of the standard lamp. This means that the illumination on the right side of the photometer head is 10 meter candles. Without moving the photometer-head carriage, place an auxiliary lamp to the left of the photometer head at such a position that a photometric balance is obtained. This, of course, means that the illumination from the left is 10 meter candles. Now connect the carriages carrying the auxiliary lamp and the photometer head by means of the tie rod provided, so that these two carriages can be moved together as a unit, but not independently. In other words, the distance d_1 is to be kept constant.

If a test lamp is put at the right end of the bench in place of the standard lamp, the candle-power of this lamp can be deter-

mined by moving the two carriages just tied together to a position where a photometric balance is obtained. The scale reading of this position of the photometer head is the candle-power of the test lamp.

Results Required:

As in Experiment 40.

EXPERIMENT 42

Photometric Measurements by Use of the Macbeth Illuminometer

Description of Instrument:

The Macbeth illuminometer consists of three main units and various accessories. The three main units are the illuminometer,

FIG. 70.—Illuminometer.   FIG. 71.—Controller.   FIG. 72.—Reference Standard.
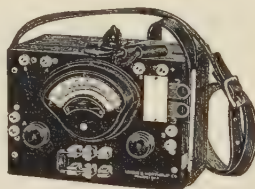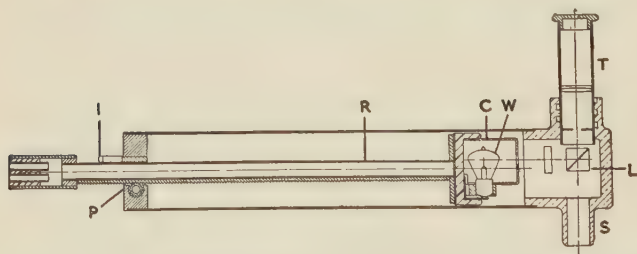
FIG. 73.—Cross Section of Illuminometer.   FIG. 74.—Cross Section of Reference Standard
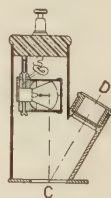
Fig. 70; the controller, Fig. 71; and the reference standard, Fig. 72. A section of the illuminometer is shown in Fig. 73.

In the rectangular head of the illuminometer is a Lummer–Brodhun cube L, mounted so that the photometric field is observed through the telescope T. Light from the surface, the brightness of which is to be measured, enters the short tube S opposite the telescope. In the large tube to which the head is attached an

electric incandescent lamp W, called the working standard, is mounted in a carriage C, with a diaphragm opening toward the cube. The carriage is attached to one end of a square rod R, and is moved up and down in the tube by means of a substantial rack and pinion P. Control of the movement and position of the working standard are positive. A direct-reading scale calibrated from 1 to 25 foot candles is engraved on one side of the rod. An index I is attached to the lower end of the tube, and is adjustable to allow for variations in filament position in different working-standard lamps. Reflection in the interior of the tube is so thoroughly eliminated that the scale follows the inverse-square law.

The second element, the controller, consists of a battery for operating the lamps, a Weston mil-ammeter, two control rheostats —one for the working standard and one for the reference standard lamp (to be described later), hereafter to be called the "reference standard"—a double-throw switch by means of which the mil-ammeter may be introduced into either the working standard circuit or the reference standard circuit. The entire controller system is enclosed in a conveniently arranged box as shown in Fig. 71. Eight binding posts are provided for completing the electrical circuits. The reference standard is connected at A, the working standard at B. The circuits are ordinarily energized by means of two No. 6 dry cells in series; these cells are inside the controller box. When these two dry cells are used they are connected electrically to binding posts marked $+$ and 2 as shown in Fig. 71. If a 4-volt battery is employed, the positive terminal is connected to the binding post marked $+$ and the negative terminal to the binding post marked 4; if a 6-volt battery is used the negative terminal is connected to the binding post marked 6 and the positive terminal as before.

Fig. 75 shows the diagram of the electrical circuit employed. When the mil-ammeter $M.A.$ in Fig. 75 is thrown from one circuit to the other a resistance R_c is automatically added to the circuit from which the mil-ammeter has been removed. This resistance R_c is equal to the resistance of the mil-ammeter, which means that current changes in the
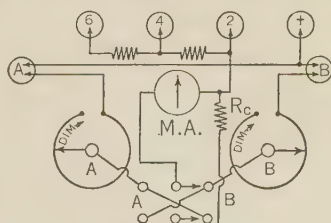
FIG. 75.

lamps are *not* produced by switching the mil-ammeter from one circuit to the other.

The third element, the reference standard, shown in Figs. 72 and 74, consists essentially of a metal housing, in which a standardized lamp is mounted. In use the reference standard is placed upon the "test plate" (to be described later) so that light from the reference standard passes through the aperture C at the bottom of the reference-standard housing and illuminates the test plate. Before the reference standard can be used it must be so seasoned and calibrated that for a specific current indicated on a certificate issued by a photometric standardizing laboratory the illumination on the test plate will have a certain value also indicated on the certificate. The illuminometer may then be made direct reading by placing its sighting aperture or short tube S, Fig. 73, into hole D of the reference standard, Fig. 74, and adjusting the current through the working standard to a value such that the observer looking into the sighting tube of the illuminometer sees the photometric field uniformly illuminated.

The test plate is a circular plate of specially prepared translucent glass which receives the illumination to be measured by the illuminometer. Very careful tests indicate that the material used for the test plate is very satisfactory for the purpose intended.

Procedure for Illumination Measurements:

Before the illuminometer or reference standard can be connected, both rheostat handles must be turned as far as they will go in the direction of the arrows pointing toward "Dim." This important preventive of carelessness cuts in resistance in the two lamp circuits and prevents passing too much current through the lamps. Connect the reference standard to the receptacle marked A and the illuminometer to the receptacle marked B, using the flexible cords provided for this purpose. Place the reference standard upon the test plate and throw the double-throw switch on the controller in the direction of A. Turn the rheostat A in the direction opposite to that indicated by the arrow, until the reading of the mil-ammeter corresponds to the current value on the reference-standard certificate.

There is now an illumination upon the test plate corresponding to the value given in the certificate. Now set the illuminometer scale to that value. Throw the double-throw switch to B and,

placing the sighting aperture of the illuminometer into the hole D in the top of the reference standard, adjust the current through the working standard by turning rheostat B up and down until a photometric balance is secured, i.e., the photometric field is uniformly illuminated. When this balance is obtained note the milammeter reading on the working standard. Now throw back the double-throw switch to the position A to see that the current through the reference standard has not changed. In each case take several settings in order to obtain average values. In actual use the working-standard current is to be kept at the value determined by this check test. To do this, keep the double-throw switch in the position B and alter the setting with rheostat B whenever the current changes. The reference lamp should be disconnected as soon as the working standard has been calibrated. It is important that the reference standard be used as little as possible in order that its calibration may be retained over a long period.

In using the illuminometer, grasp the tube in one hand, holding the telescope to the eye, and with the other hand operate either the right- or left-hand knurled head, which, moving the working standard, increases or decreases the brightness of the outer concentric field of the photometric screen. Assuming that the illuminometer has been calibrated, place the test plate at the point in the plane (horizontal or inclined) where a measurement is desired.

The plate may be placed upon a desk, table, etc., or it may be attached to the tripod and be readily moved about to the various stations desired. The construction of the tripod is such that when set up for a given height of test plate it does not require readjustment. The distance of the observer from the test plate will not introduce any error so long as the inner field of the Lummer–Brodhun cube is completely illuminated, or, in other words, the distance between the illuminometer and the test plate is immaterial, provided the distance is not so great that the surroundings of the test plate appear on the inner field of the Lummer–Brodhun cube. If the proper maximum distance is exceeded it can be readily detected, provided the test plate is not set upon a white surface, by the non-uniform illumination of the inner field. The safe maximum distance will be between 6 and 8 feet.

When making measurements, the angle between the axis of the telescope and sighting aperture and the normal to the test

plate should not exceed 30 to 40 degrees. With the equality-of-brightness type of Lummer–Brodhun screen, two concentric fields will be visible on looking through the telescope. The outer one is illuminated by the working standard, and the inner by the test plate, or other surface, the illumination upon which is to be determined. View the test plate through the telescope, turning the knurled handle on the lower end to shift the working standard nearer to or away from the cube L until a balance is secured, i.e., until the outer circle matches in brightness the inner circle illuminated by the test plate. When the surface under observation is of exactly the same color as the light from the working standard the line of demarkation between the two fields will disappear when a balance is obtained. When there is a color difference it will be impossible to obtain this disappearance. In that event, to obtain a balance, judgment is required as to when the two fields are of equal brightness. This is a matter of experience and is very quickly learned.

For those who prefer a translucent test plate viewed from beneath instead of the usual opaque test plate furnished with the illuminometer, a removable cap carrying a translucent plate is furnished. In using this attachment, which is slipped over the horn, it is advised that the illuminometer be clamped to the tripod. A new current value will be required for the working standard. For use in the determination of this value there is provided an attachment whereby the small translucent disc on the horn is attached to the bottom of the reference standard in such a manner that it receives the proper illumination from the reference-standard lamp.

When the intensity observed is too high or too low to be read directly on the scale of the illuminometer, it is necessary to use the absorbing neutral filters provided with the equipment. If the intensity observed is too high, the proper filter should be placed in the slot of the illuminometer sighting head adjacent to the short tube S of Fig. 73, that is, in the path of the light being measured; if the intensity observed is too low, the proper neutral filter should be placed in the slot located between the working-standard lamp W and the Lummer–Brodhun cube L, that is, in the path of the light from the working-standard lamp W. When using a neutral filter in the first slot mentioned above, the scale reading must be multiplied by the upper or larger figure stamped on the neutral

filter frame; when the neutral filter is in the second slot mentioned above, multiply the scale reading by the lower or smaller figure.

Procedure for Intensity Measurements:

To determine the intensity in candle-power of a lamp or any light source, set up the test plate in a plane normal to a line from the light source to the test plate, and at a measured distance away from the unknown source. Arrange suitable screens to cut off all light from the test plate except that coming directly from the source under test, or make the measurement in a dark room. Measure the illumination upon the test plate in the manner previously described. The value in foot-candles when multiplied by the square of the distance in feet from the test plate to the lamp will give the candle-power in that direction. Vertical distribution curves may be obtained from lamps by rotating them around a horizontal axis through the lamp center, or if the lamp cannot be rotated, by fastening the test plate upon an arm which may be rotated vertically about the lamp as a center. In all cases care should be taken that the light falls normally on the test plate and that the test plate is viewed through the illuminometer within the 30-degree zone normal to the test plate.

Brightness Measurements:

Brightness measurements in "apparent foot-candles" are made in a manner similar to illumination measurements, except that neither the test plate nor the translucent disc is used and the values secured either must be multiplied by the coefficient of reflection of the test plate with which the standardization has been made, or a separate standardization must be made, based on the "apparent foot-candles" emitted from the test plate in accordance with the value noted in the certificate.

If it is desired to reduce these readings to values in units of candle-power per square foot, divide by π or 3.14; for candle-power per square inch divide by $452 (3.14 \times 144)$; per square centimeter by $2920 (3.14 \times 929)$; per square meter by

$$0.292 (3.14 \times 0.0929)$$

The diameter of the field of observation is roughly one-tenth of the distance between the observer and the surface being observed. At 30 feet the diameter of the field is 3 feet.

Results Required:

To be specified at time of experiment.

PHOTOMETRIC DEFINITIONS

Candle-Power.—In the use of the illuminometer some brief explanation of photometric terms may be helpful. The candle-power is the unit in which the intensity of a source of light in any one direction is measured. The value was originally derived from the amount of light given by a sperm candle burning under certain fixed conditions, but has now been standardized so that it is an international unit.

Foot-Candle.—The foot-candle is the unit of illumination intensity, and is that intensity at a point on a surface 1 foot distant from a source of 1 candle-power. This intensity varies inversely as the square of the distance of the surface from the source. A point 2 feet from a 1 candle-power source would receive an illumination of $(\frac{1}{2})^2$, or 0.25 foot-candle.

Lumen.—The lumen is the unit of luminous flux or emanation of light from a source, and is that quantity of light which will produce a normal illumination of 1 foot-candle over a surface of 1 square foot.

Mean Spherical Candle-Power.—A source of 1 mean spherical candle-power at a distance of 1 foot from a surface normal to the incident light will deliver at all points on that surface an intensity of 1 foot-candle. A sphere of 1-foot radius with a 1-spherical-candle-power source at its center will receive an incident illumination of 1 foot-candle at all points over its inner surface. The area of a sphere of 1-foot radius is 12.57 square feet. The quantity of light emitted is therefore equal to the intensity at the surface (1 foot-candle per square foot = 1 lumen) multiplied by the area of the sphere (12.57 square feet), or quantity of light = 1×12.57 = 12.57 lumens. If the light source does not emit equal amounts of light in all directions the average is obtained and is known as the mean spherical candle-power. One mean spherical candle-power equals 12.57 lumens.

Brightness.—Brightness—termed also " intrinsic brilliancy " and " surface brightness "—refers to the appearance of a light-reflecting surface or a light source when viewed from a particular

direction. By sanction of the Illuminating Engineering Society *
and adopted by the American Institute of Electrical Engineers,
brightness is expressed in terms of candle-power per unit area.
For brightness of a high order it is usually stated in candle-power
per square inch or per square centimeter, although for reflecting
surfaces of a lower order of brightness candle-power per square
foot or per square meter has been found to result in more appreci-
able values.

Apparent Foot-Candle.—The brightness in "apparent foot-
candles" emitted from a source or surface, if distributing light
uniformly in all directions, as from a hemispherical source or from
a perfectly matte surface, when divided by π (3.14) gives candle-
power per square foot.

While "foot-candles," as ordinarily used, refers to the flux
density received on a plane, "apparent foot-candles" may be used
conveniently to express brightness † with the understanding that
"1 foot-candle of brightness would be identical in appearance to:
(a) that produced by an illumination of 1 foot-candle upon a
perfectly matte diffusing and reflecting surface of 100 per cent
reflecting power, or (b) that of a surface source of light emitting at
a density of 1 lumen per square foot, such flux being emitted in
accordance with the cosine law."

"Apparent foot-candles" becomes at once a convenient term
differing actually from the foot-candle value incident on a plane
or surface, by the absorption of that surface. A surface with
5 foot-candles incident would have an absorption of 20 per cent if
the emitted light or brightness in a given direction was four
"apparent foot-candles."

Acknowledgment:

* Trans. I. E. S. Vol. VII, p. 728.
† J. R. Cravath, *Elec. World*, Dec. 12, 1914, p. 1157.

EXPERIMENT 43

Photometric Measurements Using the
Nutting Polarization Photometer

Theory:

The optical system of the Nutting polarization photometer is shown in Fig. 76. L_1L_2 is a "twin collimator" system with its focus at S, the source of illumination. Light can enter the photometer at aperture E_1 and also at aperture E_2. Assume that parallel light enters at E_1. This light passes through the lens L_3 and then to the right-angle prism P_1, where it is reflected to the photometer cube P_2. Lens L_3 brings this light to a focus at the center of the photometer cube P_2. A plane through YY perpen-
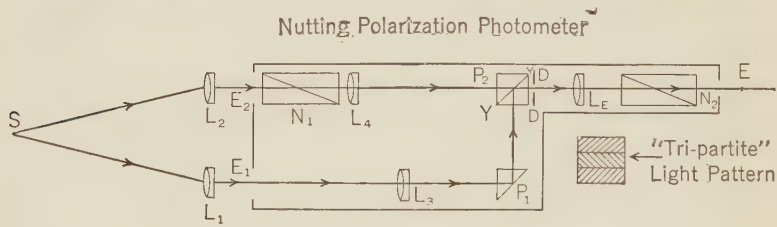
Nutting Polarization Photometer

Fig. 76.

dicular to the plane of the paper is the "plane of contact" of the two halves of the cube. The hypotenuse face of the left prism of cube P_2 is silvered with a narrow horizontal strip extending midway between the top and bottom of the cube. Some of the light that enters the cube P_2 after reflection from P_1 is reflected by the silvered strip so as to enter the "eye" lens L_E of the photometer, then passes through the rotating Nicol prism N_2 and enters the observer's eye placed at E.

Parallel light entering at aperture E_2 passes through, and is plane polarized by, the fixed Nicol prism N_1, and then passes through lens L_4. Some of this light then passes through cube P_2 over and under the horizontal silvered strip, enters the "eye" lens L_E, emerges from L_E, enters the rotating Nicol prism N_2, emerges from N_2 (if N_2 is not set for extinction), and enters the observer's eye placed at E. Lens L_4 brings this light to a focus above and below the silvered strip of cube P_2.

A diaphragm DD is placed immediately to the rear of P_2. There is a small rectangular aperture at the center of this dia-

phragm.   This aperture acts as a necessary "field of view limiting stop."

A "tri-partite" light pattern is formed on the retina of the observer's eye.   The dividing lines of this light pattern are horizontal.   The central portion of this light pattern is due to light entering the photometer at E_1; the upper and lower portions are due to light entering at E_2.   The appearance of this "tri-partite" light pattern is shown in Fig. 76.

It is important for the reader to appreciate that lenses L_3 and L_E are the objective and ocular lenses, respectively, of a telescope system for light entering aperture E_1, and that lenses L_4 and L_E are the objective and ocular lenses, respectively, of a telescope system for light entering aperture E_2.   In other words, the eye, aided by lens L_E, is viewing two images which are located in the YY plane of cube P_2.   These two images are formed by lenses L_3 and L_4 respectively.

The intensity of illumination of the upper and lower portions, but not the central portion, of the "tri-partite" light pattern can be varied by rotating the Nicol prism N_2.   If the Nicol prism N_2 is set so that its angular scale reading is 90 degrees when the intensity of the light over the path $E_2P_2L_EN_2$ is a maximum and equal to I_0, we may write

$$\frac{I_x}{I_0} = \sin^2 \theta \tag{77}$$

where θ is the angular setting of N_2 and I_x is the corresponding intensity of the light emerging from N_2.   If θ is equal to zero, the intensity of the light emerging from N_2 over the path $E_2P_2L_EN_2$ is zero and the upper and lower portions of the "tri-partite" light pattern are black.

Procedure for Measuring Per Cent Transmission through a " Light Filter ":

Arrange the optical system as in Fig. 76.   With the "light filter" *not* in position between L_1 and E_1 rotate Nicol prism N_2 so that equality of illumination in the "tri-partite" field is established, i.e., a photometric balance or "match" has been made so that the dividing lines in the "tri-partite" field have practically vanished.   Read the angular setting of N_2 and call it θ_0.   Now insert the test specimen between L_1 and E_1 and again read the

angular setting of N_2 for which a photometric balance is established. Call this angle θ_x. The per cent transmission T of the test specimen can be expressed as follows:

$$T = \frac{\sin^2 \theta_x}{\sin^2 \theta_0}(100) \qquad (78)$$

Supplementary Remarks:

The Nutting polarization photometer can be employed for many photometric measurements in addition to the one discussed under the "Procedure" above. Some other problem will be assigned when the measurement discussed above has been made.

Results Required:

To be specified at time of experiment.

SECTION VII

POLARIZED LIGHT

EXPERIMENT 44

A Study of the Properties of Polarized Light

The Simple Polariscope:

Plane mirrors M_1 and M_2 in Fig. 77 can both be rotated about a horizontal axis, and mirror M_2 can also be rotated about a vertical axis. Ordinary light can be plane polarized by reflection at
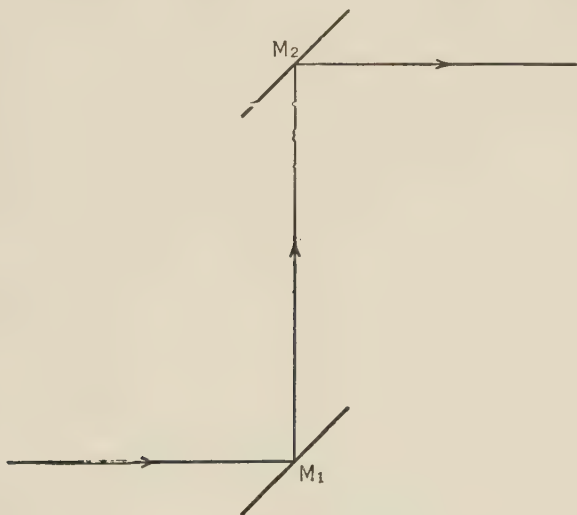
FIG. 77.

M_1 and this light can be analyzed by means of M_2, or by means of a Nicol prism which can be substituted for M_2.

Procedure:

1. *Verification of Malus Experiment by Use of Simple Polariscope. Mirrors and Nicol Prisms as Polarizers and Analyzers.*— (a) Determine the polarizing angle, using a black plane mirror M_1 as polarizer and a second black plane mirror M_2 as analyzer.

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(*b*) Remove the analyzing mirror M_2. Set the black plane mirror M_1 at the polarizing angle. Place a Nicol so as to receive the light reflected from M_1. Rotate the Nicol slowly about the line of sight through an angle of 360 degrees. Tabulate observations and state conclusions.

(*c*) Replace M_2 and set it at the polarizing angle. Remove M_1 and place the Nicol prism so that the light from the source must meet one of its end faces before arriving at mirror M_2. Rotate the Nicol prism slowly through an angle of 360 degrees. Tabulate observations and state conclusions.

(*d*) Remove M_2 and replace it by a second Nicol prism. The optical path now includes two Nicol prisms. Keep either one of the Nicol prisms stationary and rotate the other one slowly through an angle of 360 degrees. Tabulate observations and state conclusions.

2. *Determination of Plane of Polarization and Plane of Vibration of Light Transmitted through Certain Media.*—Place a glass plate upon the table of the spectrometer so that light from the collimator is incident at the glass surface at the polarizing angle. Assuming the reflecting surface of the glass plate to be vertical, the plane of incidence, and consequently the plane of polarization, is horizontal. Now place a Nicol prism between the collimator objective and the glass plate. Rotate the Nicol prism to a position such that practically no light is reflected from the glass plate; for this setting of the Nicol prism, its plane of polarization is at right angles to the original plane of incidence at the glass plate. Consequently, any vertical plane through the Nicol prism which contains incident rays is a plane of polarization of the Nicol prism.

Determine the plane of polarization of each sample supplied. Mark this plane on the Nicol prism holder, as this marked Nicol prism is to be used in Part 3.

Assuming the direction of vibration of the light waves to be at right angles to the plane of polarization, determine the direction of vibration of the light transmitted through a Nicol prism. Specify the latter direction with reference to the shorter diagonal of the end face.

3. *Determination of Plane of Polarization of Light Reflected by and Transmitted by a Pile of Glass Plates.*—Place the pile of glass plates upon the spectrometer table so that light from the collimator

is reflected by the glass plates at the polarizing angle into the telescope. Place the Nicol prism employed in Part 2 of this experiment between the pile of glass plates and the telescope and determine the plane of polarization for the light reflected from the pile of glass plates.

Now, without moving the pile of glass plates, rotate the telescope so that it will receive the light transmitted by the pile of glass plates. Place the Nicol prism between the pile of glass plates and the telescope and determine the plane of polarization for the light transmitted by the pile of glass plates.

State the relation between the plane of polarization of the reflected light and the plane of polarization of the transmitted light.

4. *Double Refraction.*—Look through the calcite crystal placed over the illuminated pinhole. With the eye vertically above the crystal, rotate the latter slowly about a vertical axis. Tabulate the observations made for a rotation of the crystal of 360 degrees.

5. *Polarization by Double Refraction.*—Set the calcite crystal so that the ordinary image and the extraordinary image are approximately of equal brightness. Look at the calcite crystal through a Nicol prism as analyzer. Rotate the Nicol slowly about the line of sight through an angle of 360 degrees, tabulate observation, and state conclusions.

6. *Refraction of Polarized Light by Calcite.*—Remove mirror M_2, Fig. 77, of the simple polariscope. Reflect light from M_1 at the polarizing angle so that it passes through a pinhole in a diaphragm placed vertically above M_1 and then through a calcite crystal placed vertically above the pinhole. Set the calcite crystal so that the ordinary image of the pinhole is seen on looking through the crystal; now set the calcite crystal so that the extraordinary image is seen. From these observations determine the principal plane of the crystal.

What conclusions can be made regarding the direction of vibration for the ordinary and extraordinary rays in a calcite crystal?

7. *Test for Strain.*—Place between crossed Nicols a piece of (a) unstrained glass, and (b) a piece of strained glass. Tabulate observations and state conclusions.

8. *Rotation of the Plane of Polarization.*—Determine the angle

through which the plane of polarization is rotated for each of the samples supplied.   State conclusions.

9. *Brewster's Law.*—Brewster's Law states that the tangent of the angle of polarization i is equal to the index of refraction n of the reflecting medium; that is,

$$\tan i = n \tag{79}$$

By the use of a spectrometer and equation (79) determine the index of refraction of each of the samples supplied.

10. *Verification of the Polarization Photometer Fundamental Equation* $\dfrac{I_x}{I_M} = \sin^2 \theta$.—For a certain definite relative angular setting of two Nicol prisms arranged "in series" the intensity of the light transmitted by the system will be a maximum.   Call this intensity I_M.   If either prism is kept stationary and the other one is rotated 90 degrees, the intensity of the light transmitted by the system will be zero.   Calling the angular setting of the rotating prism zero for the condition where the intensity of the light transmitted by the system is zero, and calling the angular setting of this same prism 90 degrees for the condition where the intensity of the light transmitted by the system is a maximum, it can be proved that

$$\frac{I_x}{I_M} = \sin^2 \theta \tag{80}$$

where I_x is the intensity of the light transmitted by the system and θ the corresponding angular setting of the rotating prism on the scale described above.

Use the Macbeth illuminometer to check equation (80) for several values of θ.

Equation (80) is the fundamental equation for certain polarization photometers and spectrophotometers.

SECTION VIII

PRISMS

Determination of Angle of Prism. Spectrometer Method 1

Procedure:

Illuminate the slit of the spectrometer. Set the prism AKL, Fig. 78, with the angle to be measured toward the collimator, dividing the beam of light, part of the beam falling on one polished face AK of the prism and part on the other polished face AL. The exact position of the prism is not important. Clamp all the movable parts of the spectrometer except the telescope. Set the telescope in the position T_1, Fig. 78, on the image of the slit reflected from the prism face AK. Read the two verniers of the divided circle. Turn the telescope to the position T_2, Fig. 78, and set on the image of the slit reflected from the face AL.
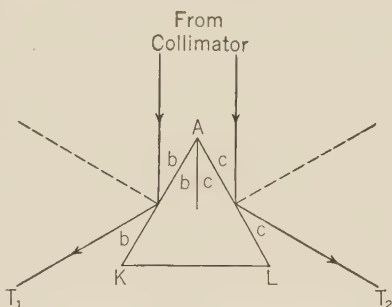
Fig. 78.

The angle through which the telescope has been turned is twice the prism angle because it has moved through the angle $b + A + c$. It is evident that the three angles b are all equal, and the same is true of the three angles c; also $b + c$ is equal to A; therefore $b + A + c = 2A$, twice the prism angle

EXPERIMENT 46

Determination of Angle of Prism. Spectrometer Method 2

Procedure:

Set the telescope of the spectrometer as near to the collimator as is convenient, and clamp all parts of the spectrometer except

the prism table. Illuminate the slit. Rotate the prism table until the image of the slit, reflected from one face AK, of the prism, Fig. 79, is set on the cross-wires of the telescope. Read the angular scale. Rotate the prism table to bring the second prism face AL, Fig. 79, into position to reflect the image of the slit to the cross-wires, and again read the angular scale.
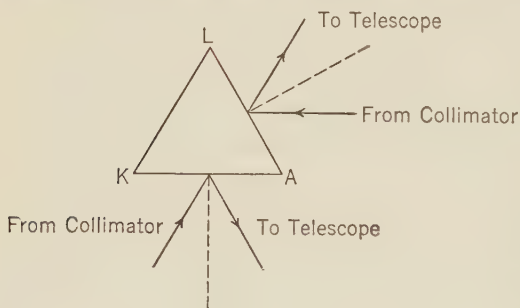
The angle through which the table has been turned is the supplement of the prism angle A in Fig. 79. In the first position the normal to one face bisects the angle between the axes of the telescope

FIG. 79.

and collimator, and in the second position the normal to the other face is turned to this same direction. The angle between the normals is the same as the angle between the two faces, but the table has clearly been turned through the supplement of this angle.

EXPERIMENT 47

Determination of Angle of Prism. Spectrometer Method 3
Method:

In this method use is made of a Gauss collimating eye-piece. The optical system of this eye-piece is shown in Fig. 80. GG' is
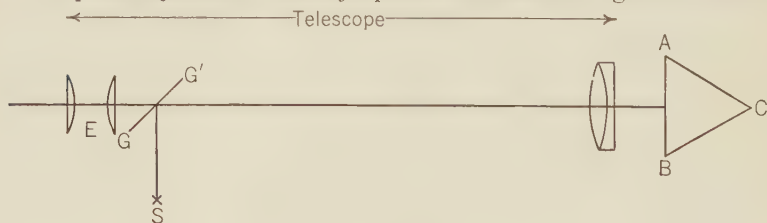
FIG. 80.

a thin glass plate set so as to make an angle of 45 degrees with the principal axis of the telescope. Light from S is incident on GG' so as to illuminate the cross-wires, and this light is then reflected by GG' in the direction of the principal axis toward the objective of the telescope.

Proceuure:

Clamp the telescope, adjust the source of light S, the collimating eye-piece E, and the prism face AB, Fig. 80, so that two images of the cross-wires are seen in the field of view of the telescope. One of these images is the "direct" image, and the other is due to light reflected from prism face AB; this second image will be called the "reflected" image. Secure coincidence between the two images by adjustments of the prism table only. Read the angular scale of the instrument. Now rotate the spectrometer table until coincidence between the direct and reflected images of the cross-wires is obtained with the light emerging from the telescope incident on face AC of the prism.

Read the angular scale of the instrument for this condition.

It can easily be shown that the difference between the two observed angular readings mentioned above is the supplement of the prism angle A.

If the table is fixed, the telescope may be turned from one coincidence to the other, in which case the light must also be moved; or the spectrometer may be turned on its base, the telescope being held during the motion. This will avoid moving the light, and may keep the telescope in a more convenient position. This method for measuring the angle between two reflecting surfaces is very precise.

<div align="center">

EXPERIMENT 48

Testing the Angles of a Right-Angle Prism

</div>

Method:

1. Consider the optical system shown in Fig. 81. ABC is the right-angle test prism with "right angle" at C. Face BC is in contact with a true plane glass surface $T.P.$, and $A.C.M.T.$ is an "auto-collimating measuring telescope." The optical arrangement of the auto-collimating measuring telescope is shown in Fig. 81. L_0 is the objective lens, E is the ocular, S is a "mil scale," that is, a linear glass scale graduated to tenths of millimeters, and P_1 is a small totally reflecting right-angle prism. The zero, or center division or line on S is longer than the other lines, so that it can be used as a single "cross-line." S is placed in the right-hand focal plane of L_0 and E is placed so that S is also in the left-hand focal plane of E. Light from a small lamp which enters the

telescope tube through a side opening A_1 is reflected by the prism
P_1 so as to illuminate the scale S.   The small lamp can be housed
in a small side tube so as to be a permanent and integral part of
the whole measuring instrument.

From the fundamental equation, $y = f\theta$, Formula 12, Section
3, of the "Miscellaneous Introductory Notes," it is obvious that
the angle subtended by a distant target can be very quickly
determined by observing the size y of its image as read on scale S
of Fig. 81, assuming that the focal length of objective lens L_0 is
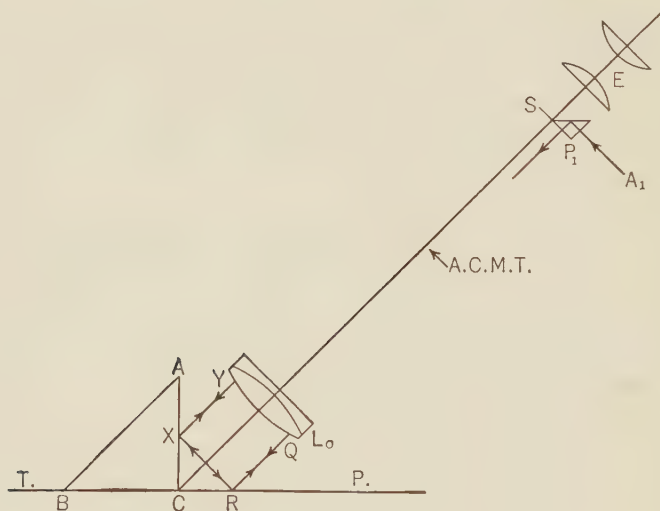
FIG. 81.

known.   When the focal length of L_0 has once been determined,
the linear mil scale S could be replaced by a scale that reads
angles directly.

It is to be noticed that the prism P_1, side opening A_1, and lamp
for illuminating S would be unnecessary where the telescope is to
be used for measuring angles subtended by distant targets.

We will now consider the problem of testing the angles of a
right-angle prism.   If the optical system shown in Fig. 81 is
properly arranged, three images of the cross-line of scale S may
be seen in the field of view of $A.C.M.T.$ One is the direct image;
the other two are formed by light which emerges from the objec-
tive of $A.C.M.T.$, is reflected by prism face AC and $T.P.$ so as to

re-enter the objective from without *A.C.M.T.* These two latter images are accounted for by the light paths *QRXY* and *YXRQ* of Fig. 81. If the angle between prism face *AC* and true plane *T. P.* is exactly 90 degrees, all three images mentioned above are superimposed.

If the error in the 90-degree angle is θ, the *angular* displacement of the two images formed by reflection at true plane *T.P.* and prism face *AC* as read on scale *S* will be 4θ.

2. A second method for testing the right angle of a prism will now be described. Consider the optical system shown in Fig. 82. *ABC* is the test prism with the right angle at *C*, and *A.C.M.T.* has the same meaning as in paragraph 1. If three images (not superimposed) of the cross-line of scale *S* appear in the field of
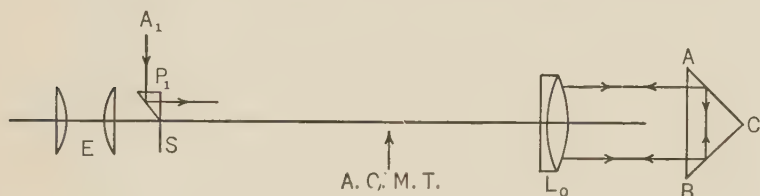
Fig. 82.

view of *A.C.M.T.*, the angle at *C* is not a right angle. If the error in the 90-degree angle is θ, the angular displacement of the two images formed by reflection as read on scale *S* will be $4n\theta$, where *n* is the index of refraction of the prism glass.

3. A method for testing the 45-degree angles of an isosceles right-angle prism will now be described. Consider the optical system shown in Fig. 83. *A.C.M.T.* has the same meaning as in paragraphs 1 and 2. *ABC* is the test prism with face *BC* in contact with the true plane *T.P.* Adjust *A.C.M.T.* so that its axis is normal to face *AB* of the prism—this will be the case when the direct image of scale *S* and the image of the same scale formed by reflection from face *AB* coincide. Now, without disturbing *A.C.M.T.*, arrange the test prism so that face *AC* is in contact with the true plane and face *AB* again receives light emerging from *A.C.M.T.* If the angles at *A* and *B* are exactly 45 degrees, the two images mentioned above will still coincide. If the two images do not coincide, the angular difference as read on scale *S*

will be 2θ, where θ is the difference between the two 45-degree angles. Assuming angle A in Fig. 83 to be larger than B,

$$A = \frac{180 - C + \theta}{2} \tag{81}$$

and

$$B = \frac{180 - C - \theta}{2} \tag{82}$$

where C is the true value of the 90-degree angle.
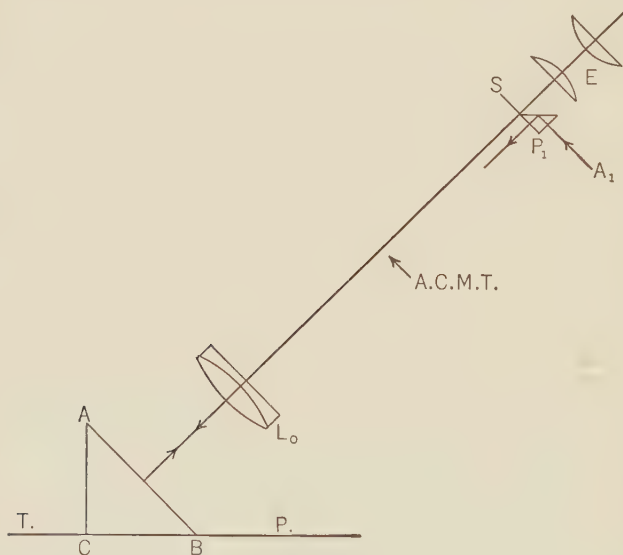
FIG. 83.

Procedure:

Obvious from discussion of "Method" above.

Results Required:

Error in 90-degree angle by Methods 1 and 2. Errors in 45-degree angles by Method 3.

EXPERIMENT 49

Measurement of the Angle of Deviation of a Penta-Prism by Use of Spectrometer

Theory:

The angle of deviation of a penta-prism is the angle between the incident direction of a ray and its emergent direction.   This
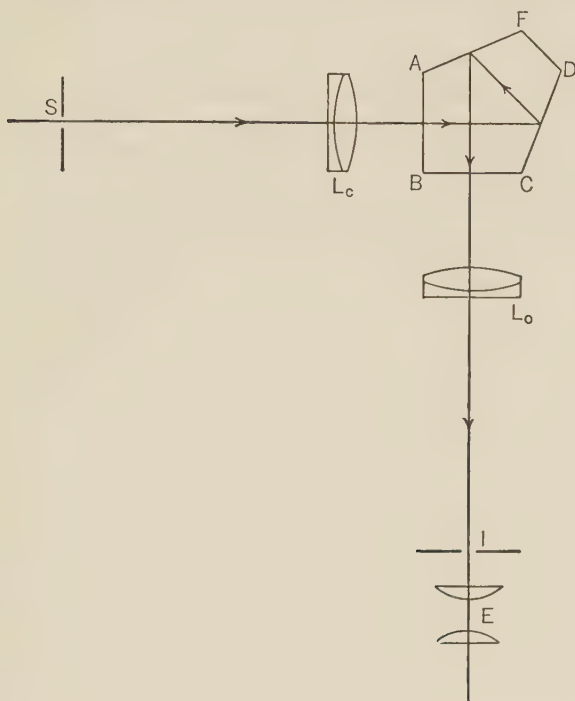
FIG. 84.

angle is intended to be 90 degrees, and any departure from 90 degrees is to be looked upon as an error in the penta-prism.

Procedure:

Arrange the optical system as in Fig. 84. S is the slit placed in the left focal plane of collimator lens L_c.   Light from the slit emerges from L_c, enters face AB of penta-prism $ABCDF$, emerges from face BC of the penta-prism, and enters the spectrometer telescope objective lens L_0. I is the image of S formed in the

focal plane of L_0. E is the eye-piece of the spectrometer telescope.
Rotate the spectrometer telescope to the position where the inter-
section of the cross-lines coincides with the slit image.   *Read the
angular position of the telescope and call this reading R_1.*

Rotate the penta-prism to the position shown in Fig. 85.
Also rotate the telescope to the position shown in Fig. 85 so that
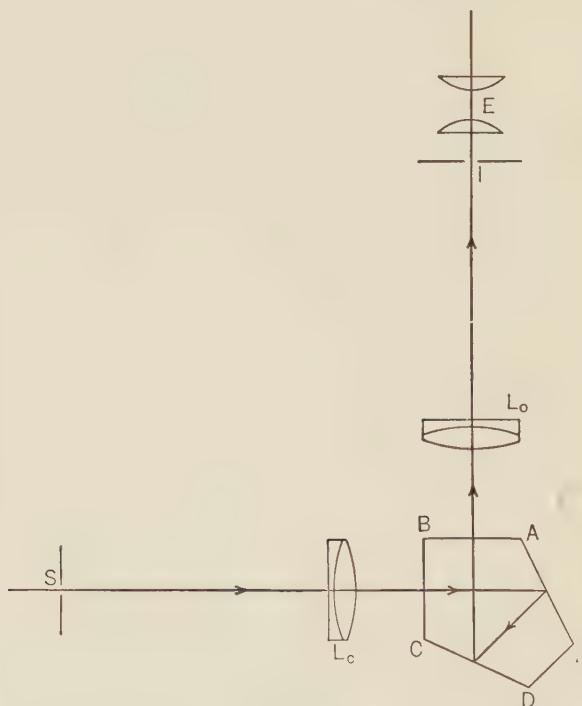
FIG. 85.

the intersection of the cross-wires coincides with the slit image.
Read the angular position of the telescope and call it R_2.

For a perfect penta-prism $R_2 - R_1$ would be equal to 180
degrees.   The difference between $(R_2 - R_1)$ and 180 degrees is
equal to twice the amount by which the angle of deviation of the
penta-prism varies from 90 degrees.

Results Required:

Angle of deviation for each prism supplied.

EXPERIMENT 50

Measurement of the Angle of Deviation of a Penta-Prism by Use of Angular Scale Collimator Target and Measuring Telescope

Preliminary Remarks:

In a perfect penta-prism the angle between an incident ray and the corresponding emergent ray is 90 degrees. This angle will be called the angle of deviation. If the angle of deviation is appreciably greater or less than 90 degrees, the penta-prism may be unsuitable for use in certain optical instruments. The object of this experiment is to measure the angle of deviation of a penta-prism in order to determine whether or not it meets the required specifications. The specifications will be supplied at the time of the experiment.

Procedure:

1. Arrange the optical system as in Fig. 86. $A.S.C.T.$ is an "angular scale collimator target" with target T in the left focal plane of collimator lens L_c. $ABCDE$ is the penta-prism being tested. Light from T emerges from the collimator lens L_c, enters the face AB of the penta-prism, emerges from face BC of penta-prism, and enters the objective lens L_0 of a measuring telescope. I is the image of T formed in the focal plane of L_0. M is a micrometer eye-piece, preferably one having a traveling "filar" or vertical wire. M is adjusted so that its measuring scale is in the same plane as I. By means of the micrometer drum on the micrometer eye-piece move the "filar" or vertical wire so that it coincides with the image of the central vertical line of target T. *Read the micrometer eye-piece scale and call this reading R_1.*

2. Now arrange the optical system as in Fig. 87. The symbols have the same meanings as in Fig. 86. $T.M.$ is a "triple mirror." The triple mirror is, in reality, a prism which has three mutually rectangular reflecting faces; it deviates any incident ray through 180 degrees. A perfect triple mirror produces the deviation of 180 degrees regardless of the way in which the light is incident on it. Light from T emerges from the collimator lens L_c, enters the face QR of triple mirror $T.M.$ from the left, emerges from face QR of triple mirror $T.M.$ from the right, enters face BC of the

penta-prism, emerges from face AB of the penta-prism, and enters the objective lens L_0 of the measuring telescope. By means of the micrometer drum on the micrometer eye-piece move the "filar" or vertical wire so that it coincides with the image of the central vertical line of target T. *Read the micrometer eye-piece scale and call this reading R_2.*
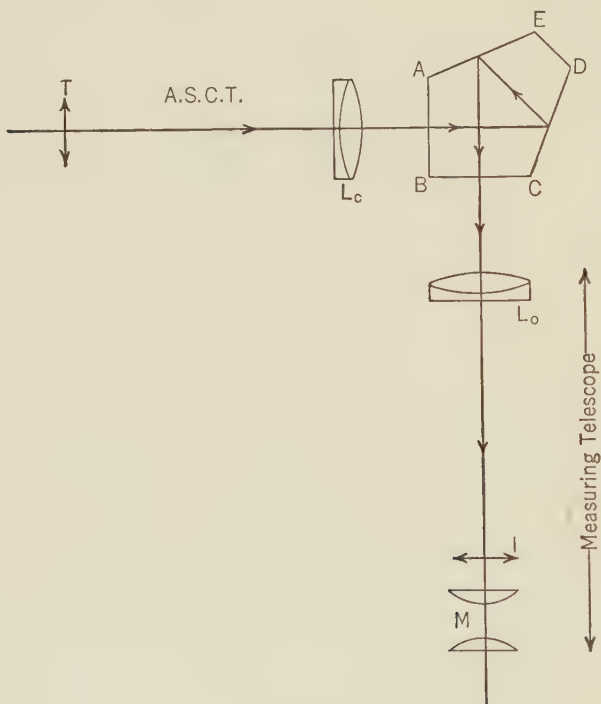
Fig. 86.

3. The difference $R_2 - R_1$ is a measure of the amount by which the angle of deviation differs from 90 degrees. This error in angular value can be determined if the focal length of L_0 is known. A sample set of data will be given.

$R_1 = 4.323$ mm. $R_2 = 4.876$ mm. $R_2 - R_1 = 0.553$ mm.

The effective focal length of L_0 is 317 mm. Each millimeter on the micrometer eye-piece scale therefore corresponds to an angle of 650.7 seconds (from the relation $y = f\theta$). The difference of

0.553 mm. between the two readings R_2 and R_1 corresponds to the *double error* in the angle of deviation of the penta-prism. Consequently, the actual error is equal to

$$0.553/2 \times 650.7 = 180 \text{ seconds}$$

4. It is now known that the angle of deviation of the penta-prism is 90 degrees $\pm$ 180 seconds. The next problem is to
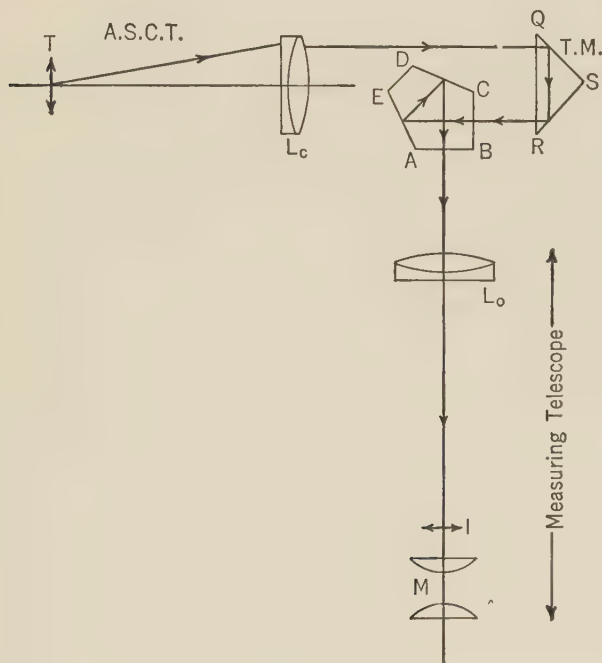
FIG. 87.

determine the algebraic sign of this error of 180 seconds. This is to be done as follows: Calculate the mean of the readings R_1 and R_2, e.g., in the above sample set of data the mean of R_1 and R_2 is equal to 4.599 mm. Arrange the optical system as in Fig. 86 and set the micrometer drum to read the mean of R_1 and R_2. If the image of the central vertical line of target T is to the right of the "filar" or vertical wire of the micrometer eye-piece the error

is plus, and conversely. For the sample set of data given above the target image was to the right. Consequently the angle of deviation of the penta-prism is 90 degrees and 180 seconds, or 90 degrees and 3 minutes.

Results Required:

Absolute value of the angle of deviation of each penta-prism supplied to within 5 seconds.

SECTION IX

SPHEROMETRY

EXPERIMENT 51

Measurement of Radius of Curvature
by Means of the Spherometer

Theory:

In its most common form the spherometer consists of a rigid metal framework provided with three pointed legs fixed at the corners of an equilateral triangle, while a fourth leg, equidistant from the other three, can be raised or lowered by means of an accurately made screw. When the instrument is placed on a true plane surface the central leg can be brought to such a position that all four legs make contact with the true plane. In this case the point of contact between the central leg and the true plane is the center of a circle and the three outer legs are making contact at points which are on this same circle. The central leg is a screw usually of $\frac{1}{2}$-mm. pitch and it is provided with a graduated disc having 50 main divisions and 500 small divisions. Thus one revolution of the disc raises or lowers the central leg by 0.5 mm., and a rotation corresponding to one of the smaller disc divisions raises or lowers the central leg 0.001 mm. At the side of the disc is a vertical scale graduated in half-millimeters, so that the number of complete revolutions of the disc can be conveniently observed.

In order to measure a radius of curvature it is necessary to obtain a reading of the instrument when all four legs are in the same plane. This can be done by placing the instrument on a true plane surface.

Assume the spherometer placed on the spherical test surface in such a manner that all four legs are in contact with the surface. Let ACE in Fig. 88 represent a section of the surface by a plane passing through the center of curvature K so that $ACEF$ forms the complete section of the sphere. C is the point of contact

between the spherical surface and the center leg of the spherometer.
Draw the diameter *CF*.   The three outer legs of the spherometer
make contact at points *E*, *A*, and a third point not shown, and
they make contact in a plane *AE* perpendicular to the paper.
The distance *DC* of the point of contact of the central leg above
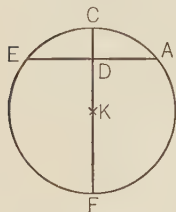
Fig. 88.

the *AE* plane is the amount by which the lower
extremity of the central leg was raised in moving
the spherometer from the contact condition on
the true plane to the contact condition on the
spherical test surface.   *DA* in Fig. 88 is the
distance from the central leg of the spherometer
to any outer leg when all four legs are in a plane.
The distance *DA* is a constant of the instrument
and is called the "radius of the base circle" of

the spherometer.   Let *DC = h* and let *DA = r*.   Call *R* the
radius of curvature of the spherical test surface.   In Fig. 88

$$CD \times DF = (DA)^2 \tag{83}$$

or

$$h(2R - h) = r^2 \tag{84}$$

or

$$R = \frac{r^2 + h^2}{2h} = \frac{r^2}{2h} + \frac{h}{2} \tag{85}$$

Equation (85) shows that the radius of curvature of the
spherical test surface can be expressed in terms of two quantities
as follows:

1. The measured value of *h*, the "vertical shift" of the center
leg in going from the contact condition on the true plane surface
to the contact condition on the spherical test surface.

2. *r*, a constant of the instrument, called the "radius of the
base circle."

The quantity *h*/2 on the right-hand side of equation (85) is
often so small compared to the quantity $r^2/2h$ that it may be
neglected.

What has been said about a convex spherical surface here
applies equally well to a concave spherical surface.

Instead of having three outer legs on a "base circle" and a
central traveling leg, it is obviously feasible to have several outer
legs on the same "base circle."   In the best spherometers, known

as ring spherometers, the three outer legs of Fig. 88 are replaced by a very accurately made ring, which makes almost a knife-edge contact with surfaces on which it is placed.   The general principle of such an instrument is the same as for the simple one discussed here.

Also in the best spherometers a more accurate method of reading the vertical shift h is employed.   In one of the very best spherometers the ring is at the top and the spherical test surfaces and the true plane are laid on the ring in turn.   A very finely divided scale is attached to the central leg.   A high-power microscope is arranged so that the observer sees in the field of view a highly magnified image of a certain scale graduation at the cross-wires.   If the central leg moves, another scale graduation appears at the cross-wires.

There are many possible spherometer arrangements in use, all of which include three essential elements, as follows:

1. A base circle, consisting of three outer legs, a ring, or the equivalent, having a known radius of base circle (r in equation (85)).

2. A central leg, which can be raised or lowered.

Fig. 89.

3. A reading device for measuring the vertical shift h in equation (85).

Figure 89 shows a common simple form of laboratory spherometer.
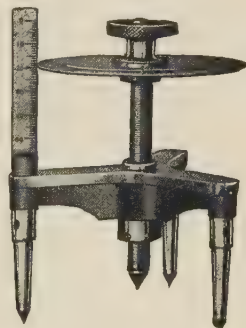
Procedure:

Measure the radius of curvature of each surface supplied with each spherometer supplied.

Results Required:

Measured radii of curvature and discussion of relative merits of the different spherometers.

SECTION X

TELESCOPIC INSTRUMENTS

EXPERIMENT 52

Arrangement of Optical Systems of Various Telescopic Instruments

Object:

The object of this experiment is to arrange and study the optical system of each of the following telescopes:

(*a*) Galileo's; (*b*) simple astronomical; (*c*) astronomical with Ramsden's eye-piece; (*d*) astronomical with Huyghens' eye-piece; (*e*) terrestrial with Huyghens' eye-piece.

Procedure:

First measure the focal lengths of all lenses supplied. For each of the telescope systems mentioned under "Object" place all lenses in the proper relative positions. Determine all possible positions of real images and cross-wires when sighting upon the test object supplied.

The erecting system of the terrestrial telescope is to consist of two converging lenses of equal focal lengths, separated by a distance equal to the focal length of either.

Results Required:

For each telescope system a diagram, with dimensions, showing the relative positions of all lenses, real images, and cross-wires.

EXPERIMENT 53

Arrangement, Study, and Measurement of Constants of a Military or Other Telescope

Procedure:

1. The optical system will be supplied unassembled. Arrange the optical system as per sketch supplied. Determine all possible positions of real images and cross-wires when sighting upon the test object supplied.

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2. Determine the magnifying power by four methods as follows:

(a) Measure diameter of entrance pupil and diameter of exit pupil. The ratio of the former to the latter is the magnifying power.

(b) View the graduated rod through the telescope with one eye and at the same time view the rod with the other eye unaided. The ratio of the number of spaces seen by the unaided eye to the number seen through the telescope in the same amount of field is the magnifying power of the telescope.

(c) Place the telescope on a divided circular table so that it can be rotated about a vertical axis. Look into the ocular and point the telescope so that the image of the vertical edge of some object a few hundred feet away is tangent to the left side of the field of the telescope. Determine the angle through which the telescope must be rotated in order to make the image of the vertical edge just mentioned tangent to the right side of the field of the telescope; this angle is called the "true field." Now invert the telescope and determine the angle as before. This angle is called the "apparent field" of the telescope. The ratio of the apparent field to the true field is the *magnifying power* of the telescope. Determine the true field also by the following method: Set up the telescope and an angular scale collimator target so that the principal axes of the two instruments coincide, the objective lens of the telescope being adjacent to the collimator lens of the angular scale collimator target. Now look into the ocular end of the telescope and read the angular extent of the collimator target image. This angular extent observed is the true field of the telescope.

(d) Measure the focal lengths of objective and ocular and the magnifying power of the erecting system. The magnifying power of the erecting system is the ratio of the size of the erect image formed by the erecting system to the inverted image formed by the objective. The magnifying power (M.P.) of the telescope can be obtained from the relation:

$$(M.P.) = \frac{f_1}{f_2}P_E \tag{86}$$

where f_1 is the focal length of objective;

   f_2 is the focal length of ocular;

   P_E is the magnifying power of erecting system.

3. Measure the eye distance with the dynameter.

4. Measure the resolving power, using the standard resolving-power test chart.

5. Obtain a smoke or chalk-dust picture of the path of the rays through the telescope when a cylindrical beam (axis parallel to principal axis) is incident on objective.

Results Required:

Values of the following constants: magnifying power, diameter of entrance pupil, diameter of exit pupil, true field, apparent field, eye distance.

EXPERIMENT 54

Testing and Measuring of Telescopes

Procedure:

1. With the eye at the ocular end look at the angular scale collimator target or any convenient distant scale graduated in degrees and read the true angular field.

2. Measure by means of the dynameter the diameter of: (a) the entrance pupil, and (b) the exit pupil. From these measurements calculate the magnifying power.

3. Measure the eye distance, using the dynameter supplied.

4. Place an illuminated mil scale in the plane of the exit pupil and on a piece of photographic paper placed as close to the objective as possible make an exposure of sufficient length to give a good print. From the print determine: (a) the diameter of the entrance pupil, and (b) the magnifying power.

5. Compare results of steps 2 and 4.

6. Observe central definition.

7. Test for color errors. See Experiment 80.

8. Measure curvature of field and astigmatism by method of Experiment 90.

9. Remove ocular and use it to project a magnified image of a convenient target on a screen. Test this image for distortion.

10. Measure resolving power.

11. Measure per cent transmission. See Experiments 86 and 87.

Results Required:

A logically arranged scientific report of findings. Conclusions.

EXPERIMENT 55

Tests, Measurements, and Adjustments of Prismatic Binoculars

Procedure:

1. With the eye at the ocular end of one barrel look at the angular scale collimator target or any convenient distant scale graduated in degrees and read the true angular field. Repeat for the other barrel.

2. Measure by means of dynameter the diameter of: (a) the entrance pupil, and (b) the exit pupil, for each barrel. From these measurements calculate the magnifying power for each barrel.

3. For each barrel measure the eye distance, using the dynameter supplied.

4. Place an illuminated mil scale in the plane of the exit pupil of one barrel and on a piece of photographic paper placed as close to the objective as possible make an exposure of sufficient length to give a good print. From the print determine: (a) the diameter of entrance pupil, and (b) magnifying power. Repeat with other barrel.

5. Compare results of steps 2 and 4.

6. Test for squareness of the Porro prisms by methods as follows:

(a) With one eye aided by one barrel of the instrument look at the collimator target; look directly, i.e., without the aid of the instrument, at the target with the other eye. If the prisms are "squared," corresponding lines in the two images formed on the retina will be parallel to each other. By means of a jeweler's screw-driver square the prisms in case they are not already squared.

(b) The method described in (a) can be carried out more conveniently by employing a small auxiliary testing telescope to assist the eye which was unaided in (a). Use this modification of (a) and compare results with those obtained under (a).

(c) Remove objective and ocular. With one eye look at the target aided only by the Porro prisms; with the other eye look at the target aided by a small auxiliary testing telescope. If the prisms are "squared," corresponding straight lines in the two images formed on the retina will be parallel to each other.

7. Test for and if necessary adjust for parallelism. Clamp

the binoculars in such a way that the target can be seen through either barrel. For either barrel center the image of the target; now look at the target through the other barrel; if the principal axes of the two barrels are parallel, the image seen in the second barrel will be centered. If the binoculars are not adjusted for parallelism, adjust the Porro prisms by means of jeweler's screwdrivers.

8. Observe central definition.

9. Test for color errors. See Experiment 80.

10. Measure curvature of field and astigmatism by method of Experiment 90.

11. Remove ocular and use it to project a magnified image of a convenient target on a screen. Test this image for distortion.

12. Measure resolving power.

13. Measure per cent transmission. See Experiments 86 and 87.

Results Required:

A logically arranged scientific report of the findings. Conclusions.

EXPERIMENT 56

Magnifying Power of Galilean or Other Telescope
Procedure:

In Fig. 90, AT is an auxiliary telescope with a mil scale in the focal plane of the eye-piece, TT is the test telescope, SS' is a linear scale placed in the focal plane of collimator lens L_c.
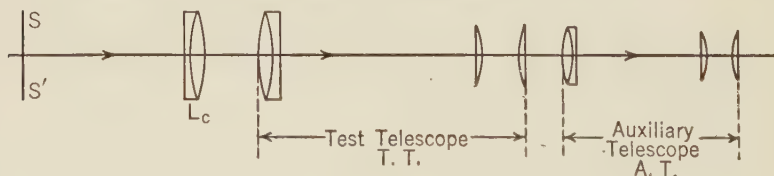
FIG. 90.

By means of the auxiliary telescope AT determine the size of image of SS' with: (a) test telescope in position, and (b) test telescope removed.

The ratio of the (a) measurement to the (b) measurement is the magnifying power of the telescope.

Note.—This method is especially suitable for the Galilean telescope, but can be employed for any telescope.

Results Required:

To be specified at time of experiment.

EXPERIMENT 57

Study of Periscope

Procedure:

1. The lenses and the prisms, but not the tube, of a standard periscope will be supplied. Line up the optical system in the proper way and determine the positions of all real images and cross-wires. Make measurements so that the relative positions of all lenses, prisms, real images, and cross-wires can be shown on a diagram.

2. Determine the true field, apparent field, diameter of entrance pupil, diameter of exit pupil, eye-distance, and magnifying power of the periscope.

Results Required:

Diagram referred to under step 1 and results under step 2.

Conclusions:

Compare optical system with that of a terrestrial telescope.

EXPERIMENT 58

Study of Bausch and Lomb Coincidence-Type Range Finder

Procedure:

1. From examination of the instrument determine the arrangement of the optical system, indicating clearly in report the function of each optical part.

Make the infinity adjustment by means of the self-contained adjusting device.

2. Determine the true field, apparent field, diameter of entrance pupil, diameter of exit pupil, eye-distance, and magnifying power of the range finder.

Results Required:

Diagram showing the optical system. Show by simple diagram and brief discussion the elementary principle of this type of range finder.

Study of Barr and Stroud Range Finder

Procedure:

1. From examination of the instrument determine the arrangement of the optical system, indicating clearly in report the function of each optical part.

2. Determine the true field, apparent field, diameter of entrance pupil, diameter of exit pupil, eye-distance, and magnifying power of the range finder.

Results Required:

Diagram showing the optical system. Show by the aid of a simple diagram and brief discussion the elementary principle of this type of range finder.

Stadimeter Test

Procedure:

(a) *Test for Setting of Infinity Mark.*—Set the range scale to any value and observe vertical target axis by looking over the stadimeter mirror and also by looking into the mirror. When the two images coincide the drum reading should be infinity. If the drum reading is not infinity for this condition, loosen the drum set screw and adjust the drum to read infinity. Check a sextant infinity reading, using the same principle.

(b) *Test for Parallelism of Mirrors.*—Observe any convenient horizontal line by reflection from the mirror and directly to the right or left of the mirror. If the horizontal line as seen directly and the mirror image of the line appear to form one unbroken line, the mirrors are parallel. If the mirrors are not parallel, adjust them so that they will be parallel.

(c) *Test for Accuracy of Range Scale.*—A special collimator target is to be used for this test. The target itself includes two vertical lines, which will be called L_1 and L_2. Set the height scale for any number in feet. Observe the two vertical lines L_1 and L_2 and adjust for coincidence. The range in yards should be the height in feet multiplied by 5. Why?

Test the range scale at about ten points rather evenly distributed from minimum to maximum.

SECTION XI

TESTING OF OPTICAL GLASS

EXPERIMENT 61

Testing Optical Glass for Flatness, Plane Parallelism, Strain, and Striae

Procedure:

(a) *Test for Flatness.*—Place the test surface on top of and in contact with a true plane surface. Clean both surfaces thoroughly. Illuminate the surface of contact from above with monochromatic light by transmission through the test specimen. Interference bands should appear over the entire surface. If the surfaces in contact are plane, the bands will be straight, parallel, and equally spaced. Pressing the surfaces will increase the width of the bands and will make it possible to detect small deviations from planeness. The interference pattern exhibited indicates the form of the test surface because any band traces the locus of all points in test surface which are at a constant distance from the true plane.

(b) *Test for Plane Parallelism.*—Place the test plate on the table of a spectrometer so that the light from the collimator is reflected into the telescope by one of the test surfaces, the telescope being set at right angles to the collimator. If the two surfaces are not parallel, two slit images will be seen, one of which revolves about the other as the plate is turned in its own plane. If the surfaces are parallel there will be but one sharply defined image of the slit.

(c) *Test for Strain.*—Place the glass test specimen between crossed Nicol prisms or between the polarizer and analyzer of any polariscope set for extinction. If the field becomes luminous, the glass test specimen is strained; if the introduction of the test specimen does not cause the field to become luminous, there is no strain present.

(d) *Test for Striae.*—Employ the method of Experiment 88.

SECTION XII

SPHERICAL MIRRORS

EXPERIMENT 62

Focal Length of a Concave Spherical Mirror and Radius of Curvature of Any Concave Spherical Reflecting Surface

Theory:

Consider the optical system shown in Fig. 91. S is a point source of illumination placed at the principal focus of collimator lens L_c. Rays emerging from L_c are incident on the concave spherical mirror M parallel to the principal axis and are reflected
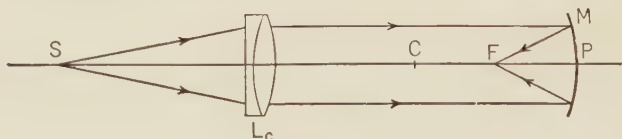
FIG. 91.

by M so as to converge to the principal focus F of the mirror. P is the pole of the mirror. C is the center of curvature of the mirror. The distance PF is the focal length of the mirror. The radius of curvature of the mirror is the distance PC and is equal to $2PF$.

Procedure:

Arrange the optical system as in Fig. 91.

Determine the position of the principal focus F by means of a small exploring screen which can be moved along the graduated track of an optical bench. Read this position of the screen and the position of the mirror.

Replace S and the collimator lens L_c by a distant target and repeat above procedure.

Results Required:

Focal lengths and radii of curvature of specimens supplied.

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<div align="center">EXPERIMENT 63</div>

Focal Length of a Concave Spherical Mirror.   Magnification Produced by a Concave Spherical Mirror

Theory:

Consider the optical system shown in Fig. 92. OA is a small illuminated target placed to the left of the principal focus F of the concave spherical mirror M.
IB is a real image of OA formed by mirror M.   The object distance $p = AP$, image distance $q = BP$, and focal length $f = FP$ of the mirror   are   connected   by
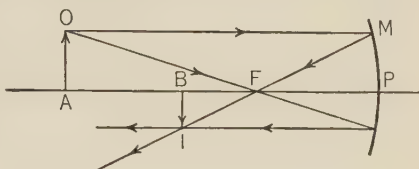
FIG. 92.

the universal mirror formula, which reads as follows:

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} \tag{87}$$

Also, the object size OA, the image size IB, the object distance p, and the image distance q are related as follows:

$$\frac{IB}{OA} = \frac{q}{p} \tag{88}$$

The ratio of image size IB to object size OA will be called the "magnification" and will be designated by m; that is,

$$m = \frac{IB}{OA} \tag{89}$$

Combining equations (88) and (89)

$$m = \frac{q}{p} \tag{90}$$

Equation (90) states the magnification is equal to the ratio of image distance q to object distance p.

Procedure:

Arrange the optical system as in Fig. 92.   Determine the position of IB by means of a small exploring screen.   For six different positions of OA measure p and q and solve for the focal

length f by means of equation (87).   Measure the object size OA and the image size IB.   Calculate m by means of equations (89) and (90) and compare results.

Results Required:

Average value of focal length and radius of curvature of concave spherical mirror.   Conclusions regarding magnification of concave mirror.

EXPERIMENT 64

Focal Length of a Concave Spherical Mirror.   Radius of Curvature of Any Concave Spherical Reflecting Surface

Theory:

In Fig. 93, S is a point source of illumination placed at the center of curvature of the concave mirror M.   Since S is at the

FIG. 93.

center of curvature, rays diverging from S strike M at normal incidence and, as a consequence, are reflected so as to retrace their original paths.   The result is that S and its image coincide.   If the mirror M is inclined slightly, S and its image S' can be seen side by side.

The focal length of a concave spherical mirror is equal to one-half its radius of curvature.

Procedure:

Obvious from above discussion.

Results Required:

Focal lengths and radii of curvature of specimens supplied.

EXPERIMENT 65

Focal Length and Radius of Curvature of a Convex Spherical Mirror

Theory:

In Fig. 94, S is a small illuminated target placed at such a position with respect to the converging lens L that the rays emerging from L will converge to point C if the convex spherical mirror M is removed.   If the rays emerging from L meet M at normal incidence these rays will be reflected by M so as to retrace

their original paths. As a result S and its image will coincide. If the mirror is inclined slightly S and its image can be seen side by side.
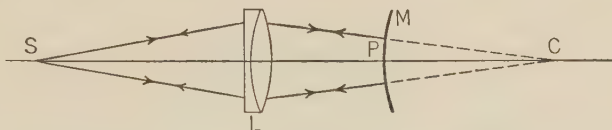
Fig. 94.

If the rays emerging from L meet M at normal incidence the point C referred to above must be the center of curvature of the convex spherical mirror M. The distance PC is the radius of curvature of the convex spherical mirror M.

Procedure:

1. Arrange the optical system as shown in Fig. 94 so that S and its image are seen side by side. This insures normal incidence at mirror M. Read the position of pole P of mirror M on the graduated track.

2. Remove the mirror M and locate the position of point C by means of an exploring screen which can be moved along the graduated track. Read the position of this exploring screen when a sharply defined image of S appears on it.

3. The difference between the two track readings is the radius of curvature of the convex spherical mirror.

Results Required:

Focal lengths and radii of curvature of convex spherical mirrors supplied.

EXPERIMENT 66

Practical Uses of the Concave Spherical Mirror

Procedure:

1. Arrange a telescopic system using a concave spherical mirror as objective.

2. Arrange a miniature search-light, using a concave spherical mirror as collimator.

3. Simulate the optical system of an infra-red spectrometer, using the necessary mirrors and prism.

4. Simulate the optical system of any other instrument employing mirrors which may be specified at time of experiment.

SECTION XIII

MEASUREMENT OF CONSTANTS OF MICROSCOPES

EXPERIMENT 67

Focal Lengths of Objectives and Eye-Pieces
of a Compound Microscope

Theory:

The principle of the method to be employed in determining small effective focal lengths can be made clear by reference to Fig. 95. OA represents an object placed to the left of the first principal focus F_1 of test lens L_x. E_1 and E_2 are the first and
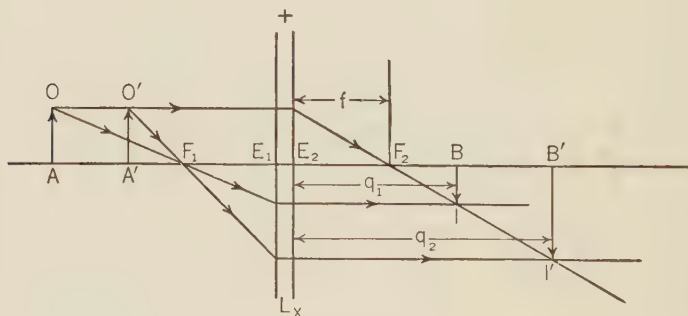
FIG. 95.

second principal points, respectively, of L_x. F_2 is the second principal focus of L_x. IB is the real image of OA. The magnification M_1 can be expressed as follows:

$$M_1 = \frac{IB}{OA} \tag{91}$$

From the geometry of the figure

$$\frac{IB}{OA} = \frac{q_1 - f}{f} \tag{92}$$

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where $f = E_2F_2$ is the focal length of L_x and $q_1 = E_2B$ is the image distance.

Combining equations (91) and (92)

$$M_1 = \frac{q_1 - f}{f} \tag{93}$$

Now assume object OA moved to a new position indicated by $O'A'$. The corresponding image is $I'B'$ in Fig. 95. The new magnification M_2 can be expressed as follows:

$$M_2 = \frac{I'B'}{O'A'} = \frac{q_2 - f}{f} \tag{94}$$

Combining equations (93) and (94)

$$f = \frac{q_2 - q_1}{M_2 - M_1} \tag{95}$$

Equation (95) forms the basis of this method for measuring short focal lengths. The numerator of equation (95) represents the amount by which the image shifts, and the denominator contains the two magnifications corresponding to images IB and $I'B'$ in Fig. 95.

Procedure:

Arrange the optical system as in Fig. 95. OA is a linear scale ruled on glass and graduated to tenths of a millimeter. By any convenient means measure the size of a magnified image IB corresponding to a known length of scale OA. From these data calculate M_1 in equation (95). Read the position of image IB. Move the graduated scale to a new position $O'A'$. Calculate M_2 and read the position of image $I'B'$.

The distance between the first image IB and the second image $I'B'$ is the numerator of equation (95). M_1 and M_2 have been determined. Hence f can be calculated.

Repeat the above observations for three or four other combinations of $q_2 - q_1$ and $M_2 - M_1$.

Note.—If the microscope draw-tube is graduated, the quantity $q_2 - q_1$ in equation (95) can be determined by means of the draw-tube scale.

A convenient means for measuring the image sizes is a standard micrometer eye-piece, which can be put in place of the regular

eye-piece of the microscope. This micrometer eye piece has a range of 10 mm. and can be read to 0.01 mm.

From the last two paragraphs it should be obvious that very little apparatus other than the microscope is required for this experiment.

Results Required:

Focal lengths of all objectives and eye-pieces of the microscope supplied.

EXPERIMENT 68

Measurement of Magnifying Power of a Compound Microscope

Procedure:

(A)

Arrange the optical system as in Fig. 96. M is the test microscope. S_1S_2 is a small graduated scale ruled on glass. A scale having adjacent graduations 0.1 mm. apart is usually satisfactory. T_1T_2 is a comparatively large linear scale placed 250 mm. from the observer's eye E, that is, at the distance of distinct vision for a normal eye.
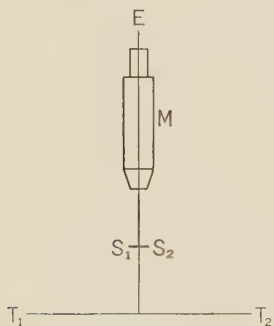
Fig. 96.

Focus the microscope on the small scale S_1S_2. With one eye look at S_1S_2 through the microscope, and with the other eye look directly at the large scale T_1T_2. Two superimposed images will be formed on the retina. The retinal image of 1 mm. of S_1S_2 may be superimposed on the retinal image of 100 mm. on T_1T_2. The magnifying power of the microscope in such a case is 100. For the superimposed retinal images determine the length of T_1T_2 corresponding to a known length of S_1S_2. The ratio of the former to the latter is the magnification of the microscope. Measure six corresponding pairs of retinal images.

(B)

Arrange the optical system as in Fig. 97 and repeat observations made in (A).

In Fig. 97 G_1G_2 is a thin plate of glass inclined at 45 degrees

to the principal axis of microscope M. S_1S_2 and T_1T_2 have the same meanings as in Fig. 96. The observer's eye at E is placed 250 mm. (distance of distinct vision for the normal eye) from T_1T_2. Two superimposed images are formed on the retina. One is a magnified image of S_1S_2 due to light which, on emerging from M, is reflected by the glass plate G_1G_2 into the observer's eye; the second is an image of scale T_1T_2, due to light which travels from T_1T_2 *through* the glass plate G_1G_2 directly into the observer's eye. As in method (A) above, determine the length of T_1T_2 corresponding to a known length of S_1S_2. The ratio of the former to the latter is the magnification of the micro-

Fig. 97.

scope. Measure six corresponding pairs of retinal images.

Results Required:

Magnifications of microscope for combinations of objective, tube length, and eye-piece to be specified at time of experiment.

EXPERIMENT 69

Measurement of Numerical Aperture $(N.A.)$ of a Compound-Microscope Objective

Theory:

The resolving power of an optical instrument is its ability to form separate and distinct images of adjacent elements of an object, these elements being very close together. In the use of the microscope, resolving power is often of more importance than magnification because of the necessity of observing fine details. The measure of the resolving power of a microscope is the so-called "Numerical Aperture" $(N.A.)$ of the objective. The greater the value of the $N.A.$ the higher the resolving power of the objective, and, consequently, the finer the detail which the instrument will show. Numerical Aperture $(N.A.)$ is defined by the formula

$$N.A. = n \sin u \qquad (96)$$

where n is the index of refraction of the medium immediately
outside the objective;

u is half the angular aperture of the objective.

$N.A.$ varies from about 0.08 for low-power objectives to about
1.3 for high-power objectives.

Procedure:

Arrange the optical system as in Fig. 98. M is a compound
microscope with the test objective at the right end. Adjust the
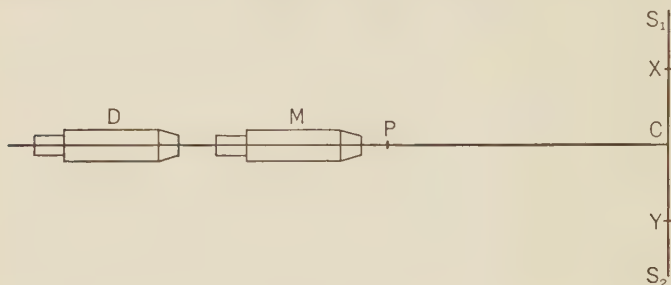
FIG. 98.

tube length to the standard distance, namely, 160 mm. P is the
position that any small object must occupy if a sharply defined
image of such object is to be seen on looking into the ocular end
of the microscope. S_1S_2 is a linear graduated scale placed per-
pendicular to the principal axis of the microscope. D is a measur-
ing microscope or a positive eye-piece such as the Ramsden's.
Focus D on the exit pupil of microscope M. Now move any con-
venient object, such as a pencil, along the scale S_1S_2 so that it can
just be seen at the extreme left edge of the exit pupil by the aid of
D. Note this reading on scale S_1S_2. This reading corresponds
to point X in Fig. 98. Repeat for the extreme right edge of the
exit pupil. This reading corresponds to point Y in Fig. 98. The
angle u of equation (96) can now be found from the relation

$$\tan u = \frac{XY}{2PC} \qquad (97)$$

The distance XY has already been determined. Now measure
the distance PC. Assuming the medium immediately outside the
objective to be air, calculate the $N.A.$ by use of equation (96).

Results Required:

N.A. for objectives supplied.

Measurement of Magnification of a Compound Microscope

Procedure:

1. Adjust the tube length to 160 mm. Place a small linear scale on the stage of the microscope. A convenient scale for this purpose is one ruled on glass and graduated to tenths of a millimeter. A real enlarged image of a portion of this scale will be formed by the objective of the microscope. Remove the regular eye-piece of the microscope and by any convenient means measure the size of the image corresponding to any known length of the scale itself. The ratio of the length of the image of the scale formed by the objective to the length of the corresponding portion of the scale itself is the "objective magnification" or "initial magnification."

A convenient means for measuring the image size is a standard micrometer eye-piece, which can be put in place of the regular eye-piece of the microscope. This micrometer eye-piece has a range of 10 mm. and can be read to 0.01 mm.

2. Measure the effective focal length of the microscope eye-piece by a method considered suitable. The "eye-piece magnification" is equal to $250/E.F.$ where $E.F.$ is the effective focal length of the eye-piece in millimeters.

3. The "microscope magnification" is equal to the "objective magnification" multiplied by the "eye-piece magnification."

Results Required:

Microscope magnifications for various combinations of objective, eye-piece, and tube length.

SECTION XIV

MISCELLANEOUS LENS TESTS

EXPERIMENT 71

Determination of Variation of Depth of Focus of a Telescopic or Photographic Objective Lens System with: (a) Effective Focal Length, (b) Relative Aperture (Speed), and (c) Object Distance

Theory:

A well-corrected lens images only one plane of the object space sharply. A lens focused on a building, for example, images at the same time with satisfactory sharpness objects in front of the building or trees to the rear because a small decrease in definition produced by deliberately throwing the lens "out of focus" by a small amount is not noticeable to the eye. If, instead of point-for-point imagery, the image corresponding to each point of the object is a "circle of confusion" of diameter 0.1 mm., then this image viewed at the distance of distinct vision for the normal eye (250 mm.) will appear as a point; consequently, an image will appear sharp if all points in the object are imaged by circles of confusion of diameters not exceeding 0.1 mm.

Assume now that a target is placed at a known distance from a lens and that the lens is racked so as to produce a sharply defined image of the target on a screen. For the speed and object distance at which the lens is operating, the object distance can be increased or decreased a certain amount without producing a noticeable change in the image quality. The distance between the extreme positions satisfying the condition just mentioned is the "depth of focus" for the given object distance and speed at which the lens is operating.

It is shown in treatises on lens optics that, *regardless of the type of construction*, the depth of focus is inversely proportional to:

(a) Effective focal length.
(b) Relative aperture (speed).
(c) Reciprocal of object distance.

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From the above statement it is at once evident that high speed, large effective focal length, and great depth of focus are incompatible. This is an unalterable experimental fact, and as a consequence the lens designer cannot make depth of focus subject to special correction. In any lens great speed can be obtained only at a sacrifice of depth of focus, and conversely.

Procedure:

Two lenses will be supplied—one of comparatively long focal length, and the other of comparatively short focal length. Place the target at such a position that the object distance is 100 feet. The test lens will be mounted on the lens board of a "camera obscura."

1. For maximum relative aperture focus either lens so that a sharply defined image is formed on the ground glass of the camera. Use a magnifying glass in order to increase the accuracy of this setting. Now without disturbing the focus determine the maximum and minimum object distances for which no noticeable change in the distinctness of the image occurs. Repeat at speeds of $f/11$, $f/16$, $f/22$, $f/32$, and $f/45$.

2. Repeat step 1 at distances of 50, 25, 15, 12, 10, 8, and 6 feet, respectively.

3. Repeat steps 1 and 2 with the second lens.

Results Required:

Results arranged in tabular form to show how depth of focus varies with:

- (a) Effective focal length.
- (b) Relative aperture (speed).
- (c) Reciprocal of object distance.

EXPERIMENT 72

Calibration of the Relative Aperture or Speed Scale of a Camera Lens

Theory:

Assume a point source of illumination at a distance p from a lens of effective aperture diameter D. The quantity of light Q entering the lens is directly proportional to the area of the effective

lens aperture and inversely proportional to the square of the object distance p; that is,

$$Q = c\frac{D^2}{p^2} \tag{98}$$

where c is a constant.

Now assume the source of illumination to have finite dimensions with an area A. The quantity of light Q entering the lens is

$$Q = \frac{kD^2A}{p^2} \tag{99}$$

where k is a constant.

Neglecting losses in the lens system, the brightness B of the image of this object of finite dimensions is

$$B = \frac{kD^2A}{p^2a} \tag{100}$$

where a is the area of the image.

A fundamental proposition in geometrical optics reads as follows:

$$\frac{A}{a} = \left(\frac{p}{q}\right)^2 \tag{101}$$

where A, a, and p have the meanings given above, and q is the image distance.

Substituting equation (101) in equation (100)

$$B = k\left(\frac{D}{q}\right)^2 \tag{102}$$

For a distant object, $q = f$, where f is the focal length of the lens. In this case

$$B = k\left(\frac{D}{f}\right)^2 \tag{103}$$

Equation (103) states that the brightness of the illumination of the image of a distant object is proportional to the square of the ratio of the aperture diameter to the focal length.

In photographic work it is necessary to be able to vary the effective aperture diameter of the camera lens in order to obtain satisfactory results for various subjects being photographed under various conditions. If a well-defined picture is to be obtained of

a train traveling 60 miles per hour at right angles to the principal axis of the camera lens, the time of exposure must be very short (probably 0.001 second or less), and consequently the lens must be used "wide open," in order to get enough light to the negative to produce the proper photo-chemical action.   In other cases the time of exposure can be much larger, and consequently the lens can be "stopped down" so as to give greater depth of focus and better definition.

Practically all photographic lenses are provided with an "Iris" diaphragm, i.e., a device by means of which the aperture diameter of the lens can be varied.   In photography this diaphragm is called a "stop."

The proper duration of exposure varies inversely with the brightness of the image.   Let us assume that we wish to arrange a series of stop values such that the brightness of the image can be varied from a maximum to a minimum by fractional increments. If D/f in equation (103) is given the value unity, the brightness will have a certain value, which we will call B_1.   If D/f is given the value $1/4$, the brightness will be $B_1/16$.   Column 2 of Table I shows the brightness values B for different values of D/f.   The maximum value of D/f for ordinary camera lenses is $1/4$, and consequently the last three columns of Table I are based on $D/f = 1/4$ taken as unity.

The fifth column of Table I, headed "Relative Time of Exposure," shows, for example, that the same amount of light energy will reach the negative for a 1-second exposure when the aperture diameter D is one-fourth the focal length as for an 8-second exposure when the aperture diameter D is $1/11.3$ part of the focal length.

The numbers in the third colum of Table I are called f numbers; each f number represents the number of times the diameter of the stop is contained in the focal length.   A low f number represents a high relative aperture or high relative speed, and conversely. If a lens is stopped so that the f number is 7.7 it is common practice to say that the lens is "working at f 7.7."

A system of stop numbers, called the "uniform system," in which the stop numbers are proportional to the necessary time of exposure, is employed to some extent.   This system is often called the " U. S. system."   Column 6 of Table I gives the U. S. stop numbers corresponding to the f numbers of column 3.

TABLE I

$\dfrac{D}{f}$	B	f Number	Relative Brightness or Illumination	Relative Time of Exposure	U. S. Stop Number
$\dfrac{1}{4}$	$\dfrac{B_1}{16}$	4.0	1	1	1
$\dfrac{1}{5.65}$	$\dfrac{B_1}{32}$	5.65	$\dfrac{1}{2}$	2	2
$\dfrac{1}{8}$	$\dfrac{B_1}{64}$	8.0	$\dfrac{1}{4}$	4	4
$\dfrac{1}{11.3}$	$\dfrac{B_1}{128}$	11.3	$\dfrac{1}{8}$	8	8
$\dfrac{1}{16}$	$\dfrac{B_1}{256}$	16.0	$\dfrac{1}{16}$	16	16
$\dfrac{1}{22.6}$	$\dfrac{B_1}{512}$	22.6	$\dfrac{1}{32}$	32	32
$\dfrac{1}{32}$	$\dfrac{B_1}{1024}$	32.0	$\dfrac{1}{64}$	64	64
$\dfrac{1}{45.2}$	$\dfrac{B_1}{2048}$	45.2	$\dfrac{1}{128}$	128	128
$\dfrac{1}{64}$	$\dfrac{B_1}{4096}$	64.0	$\dfrac{1}{256}$	256	256

Procedure:

1. Measure the diameter of the stop of the camera lens supplied corresponding to several f numbers as read from the f number scale. For each diameter measured and corresponding f number, calculate the effective focal length of the lens and average the results.

2. Measure the effective focal length of the lens by a precision method. From the measured values of the diameter correspond-

ing to the different f numbers, determine the error on the f number scale for each case.

3. A special target consisting of a uniformly illuminated white surface will be provided. Set up the camera in such a manner that a sharply defined image of this target is formed on the focusing screen of the camera. By means of the photometer provided, measure the illumination on the screen corresponding to several different f numbers.

Examine the data to determine whether the illumination varies according to Table I.

SECTION XV

THE INVISIBLE SPECTRUM. INFRA-RED AND ULTRA-VIOLET SPECTROMETRY

EXPERIMENT 73

Infra-Red Spectrometry. Determination of: (a) Emission Curve of a Source of Radiation in the Infra-Red Region, and (b) Per Cent Transmission Curves of Certain Materials in the Infra-Red Region

Theory:

The emission curve for a given source of radiation in the infra-red region co-ordinates wave length in the infra-red region on the x axis and rate of emission of radiant energy on the y axis.

The per cent transmission curve for a certain material in the infra-red region co-ordinates wave length in the infra-red region on the x axis and per cent transmission on the y axis.

The optical system of the infra-red spectrometer is shown in Fig. 99. The illuminated slit S_1 is placed at the focus of a gold-plated concave mirror M_1. M_1 therefore acts as a collimator, i.e., rays incident on M_1 diverging from S_1 are reflected by M_1 as parallel rays and meet the face AB of the rock-salt prism ABC. These rays are refracted and dispersed by prism ABC and emerge from the prism, are then incident on the gold-plated plane mirror M_2, where they are reflected so as to meet the gold-plated concave mirror M_3. The rays incident on M_3 are still parallel to each other, and consequently are reflected by M_3 so as to converge to the principal focus of M_3 located in the plane of the slit S_2. A very sensitive thermo-pile is placed at the focus of M_3. This thermo-pile is connected to a galvanometer system.

A mechanical arrangement is provided at D whereby the rock-salt prism can be rotated about an axis normal to the plane of the paper. This means that the radiation corresponding to only one wave length will be focused on the thermo-pile at one time. Radiation corresponding to other wave lengths is focused at points on

either side of the thermo-pile slit, and consequently does not affect the thermo-pile. There is a wave-length scale at D, so that the wave length corresponding to the radiation received at the thermo-pile can be read.

The magnitude of the galvanometer deflection produced is taken as a measure of the rate at which radiant energy corresponding to a given wave length is being received at the thermo-pile.
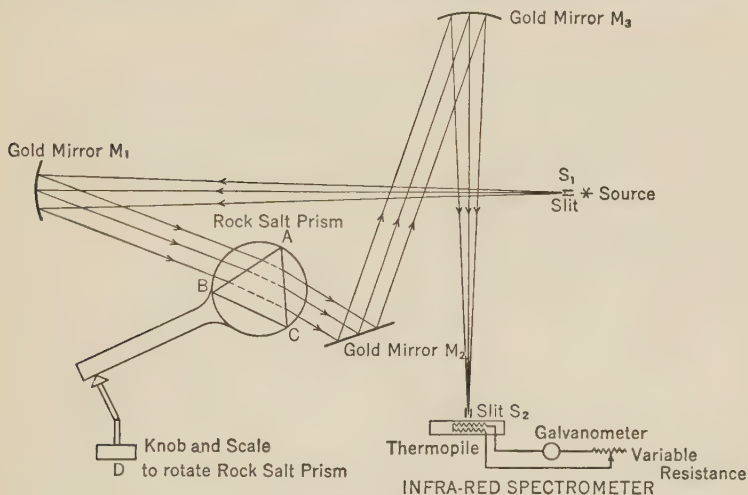
FIG. 99.

Procedure:

1. Place in front of slit S_1 the source of radiation specified. Set the wave-length drum to read 1.0μ and read the galvanometer deflection produced. Repeat this procedure for several other values of wave length up to 9.0μ. From the data obtained, plot a curve with wave lengths as abscissae and galvanometer deflections as ordinates. This curve will be called the emission curve for the source tested. The ordinates of this curve are relative and not absolute values—the curve shows the spectral distribution of the radiation from the source.

2. Place the absorption sample supplied (probably a piece of Corning colored glass) between the rock-salt prism and the gold-plated plane mirror M_2. Set the wave-length drum to read 1.0μ and read the galvanometer deflection produced. Call this deflection d_1.

Now remove the absorption sample and read the galvanometer deflection produced. Call this deflection d_2. The ratio of d_1 to d_2 is the per cent transmission at 1.0μ for the particular absorption sample used. Repeat this procedure for several other values of wave length up to 9.0μ. From the data obtained plot a curve with wave lengths as abscissae and per cent transmissions as ordinates. This curve will be called the "per cent transmission curve."

Remarks:

Adjustments of the optical system, the slits, and the galvanometer system will be made by the instructor. The student is advised not to attempt to make these adjustments without consulting the instructor.

Results Required:

To be specified at time of experiment.

EXPERIMENT 74

Measurements in the Ultra-Violet Region by Use of a Quartz Spectrograph

Theory:

Figure 100 shows the optical system of the quartz spectrograph. C is the collimator tube with slit S at the principal focus
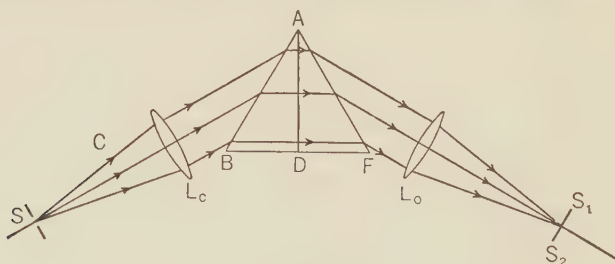
Fig. 100.

of quartz collimator lens L_c. $ABDF$ is the Cornu quartz double prism, L_0 is the quartz objective lens of the "telescope," and S_1S_2 represents a fluorescent screen or photographic plate as detector of the ultra-violet radiation. The quartz lenses L_c and L_0 are simple lenses, and consequently are not achromatic. The collimator is adjusted so as to make the rays in the middle of the spec-

trum traverse the prism parallel to each other. The extreme ultra-violet rays are therefore convergent and the red rays are divergent. On account of this chromatism (about 13 per cent), it is necessary that the photographic plate at S_1S_2 be inclined at an angle of about 20 degrees to the normal to the principal axis of L_0.

A direct-reading wave-length scale of range 200mμ to 800mμ is placed in the plane of S_1S_2 so that the wave length of any fluorescing line can be read.

Procedure:

By use of the fluorescent screen at S_1S_2 determine the ultra-violet spectrum due to:

(a) The sun.

(b) The sun filtered by ordinary window glass.

(c) The radiation from the quartz-tube mercury-vapor lamp.

(d) The radiation from the quartz-tube mercury-vapor lamp filtered by a sheet of plate glass.

(e) Any other sources specified.

Repeat (a), (b), (c), (d), and (e), using a photographic plate instead of the fluorescent screen.

Results Required:

Conclusions from observations on fluorescent screen and from photographic plates.

<div align="center">EXPERIMENT 75</div>

Measurements in the Ultra-Violet Spectrum by Use of a Constant-Deviation Quartz Wave-Length Spectrometer or Ultra-Violet Monochromatic Illuminator

Theory:

Figure 101 shows the optical arrangement of the constant-deviation quartz wave-length spectrometer or ultra-violet mono-chromatic illuminator. K is the collimator with slit at S_1 and quartz lens L_c as collimator lens, M is a plane mirror, $ABDC$ is the Cornu double-quartz prism, T is the telescope with quartz objective lens L_0 and ocular slit S_2. Plane mirror M and prism $ABDC$ are rigidly mounted on a prism table. This prism table can be rotated through a small angle about an axis perpendicular to the plane of the paper. This rotation is produced by means of a drum graduated to read directly in wave-length units in the ultra-violet spectrum.

For any given angular setting of the prism table carrying M and $ABDC$ monochromatic rays corresponding to some one wave length in the ultra-violet spectrum emerge from prism face AC in a direction parallel to the principal axis of L_0; for any other angular setting of M and $ABDC$ monochromatic rays corresponding to another wave length emerge from AC in a direction parallel to the principal axis of L_0. The axes of K and T are fixed with respect to each other and the angle between them is 90 degrees.
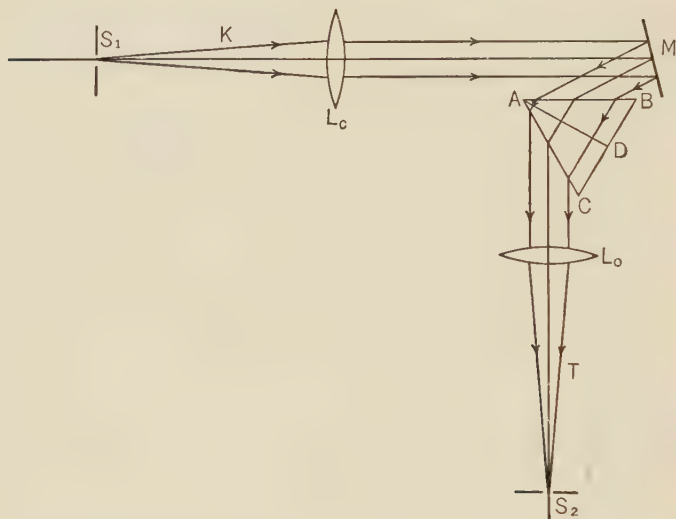
Rays emerging from $ABDC$ and incident on L_0 in a direction parallel to the principal axis will converge to the principal focus of L_0. The slit S_2 is placed in the focal plane of L_0 and its width is adjusted so that radiation from S_1 corresponding to only one wave length comes to a focus between the jaws of the slit.

A small fluorescent screen is placed at the focus of L_0 between the jaws of the slit S_2; this fluorescent screen is used as the detector of the ultra-violet radiation, which is brought to a focus at S_2.

The lenses L_c and L_0 are simple quartz lenses, i.e., they are not achromatic. This means a variation in focal length of several per cent for the wave-length range of the instrument. In order for L_c to act as collimator and L_0 to act as objective for different wave lengths, the distance between S_1 and L_c and the distance

between L_0 and S_2 must be adjustable. L_c and L_0 are movable. An index and scale are provided for both L_c and L_0, so that both of these lenses can be placed in their proper positions relative to S_1 and S_2 respectively for any wave length.

In order for radiation corresponding to any arbitrarily chosen wave length in the ultra-violet region to come to a sharp focus at S_2 it is necessary to set three wave-length scales to read the wave length desired.   These three scales are as follows:

1. The wave-length scale on the drum that rotates M and $ABDC$.

2. The wave-length scale along which L_c moves.

3. The wave-length scale along which L_0 moves.

Procedure:

Illuminate slit S_1 by means of a quartz-tube mercury-vapor lamp.   Determine the wave lengths of all the lines in the ultra-violet spectrum of mercury between 0.2μ and about 0.4μ.   This is to be done by "exploring" the spectrum, i.e., by changing the wave-length scales slowly and carefully from one end of the range to the other and noting those wave lengths at which fluorescence occurs at S_2.

Additional Measurements:

To be specified at time of experiment.

Results Required:

To be specified at time of experiment.

SECTION XVI

SPECTROPHOTOMETRY

EXPERIMENT 76

Spectrophotometric Comparison of Two Sources of Illumination by Use of Brace-Lemon Polarization Spectrophotometer

Theory:

The optical system of the Brace–Lemon polarization spectrophotometer is shown in Fig. 102. C_1 is a collimator having a vertical slit S_1 in the focal plane of collimator lens L_1. C_2 is a collimator having a vertical slit S_2 in the focal plane of collimator lens L_2. Collimator system C_2 also includes a stationary Nicol prism N_1 and a rotating Nicol prism N_2. This means that N_1 acts as a polarizer and N_2 as an analyzer, and also that the intensity of the light emerging from L_2 can be varied from a certain maximum to zero by rotating N_2. $ABCD$ is a so-called Brace prism. This prism is equiangular and consists of two exactly similar right-angle prisms cemented together in a plane passing through the edge BB', Fig. 103. Before the two right-angle prisms are cemented together a horizontal strip of silver is deposited on one of them. The two edges of this silver strip, as shown in Fig. 103, are parallel to the line of separation BD of the two right-angle prisms, and the strip is located midway between the base and top of the prism.

Light from S_1, Fig. 102, emerges as parallel light from L_1, enters the face CD of the prism $ABCD$, passes over and under the horizontal silver strip of the prism, emerges from face AB of the prism, and enters the objective lens L_3 of the observing tube T. As a result, two narrow horizontal spectra of the source illuminating S_1 are formed in the focal plane of L_3. These two spectra can be observed by looking into an ocular system placed in back of the ocular slit S_3.

Light from S_2 passes through N_1 and N_2 (if N_2 is not set for

extinction), emerges from L_2 as parallel light, enters face AD of prism $ABCD$, is reflected from the horizontal silver strip of the prism, emerges from face AB, enters the objective lens L_3 of the observing tube T. As a result, a narrow horizontal spectrum of the source illuminating S_2 is formed in the focal plane of L_3. This spectrum can be observed by looking into an ocular system placed in back of the ocular slit S_3. If S_1 and S_2 are both illuminated at the same time, the two spectra just mentioned will appear in the

Brace-Lemon Polarization Spectrophotometer

Fig. 103.

"Tri-partite" Pattern

From S_1 From S_2
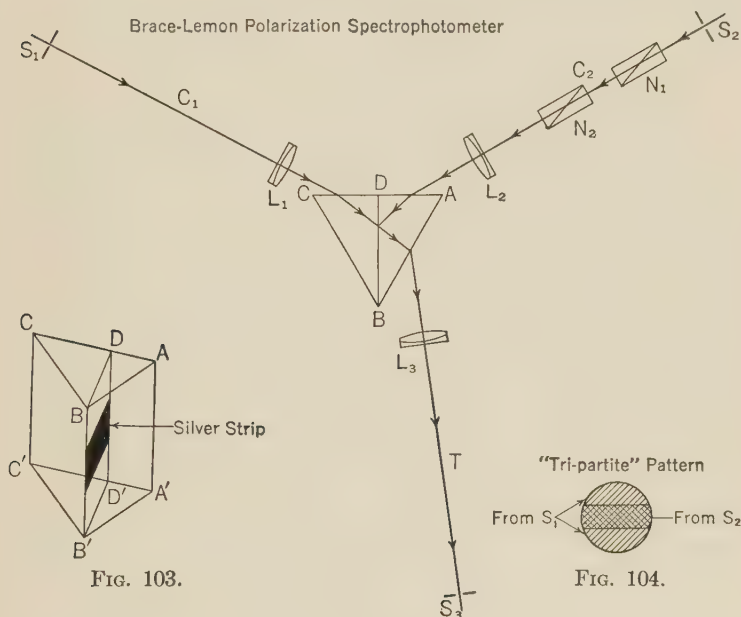
Fig. 104.

Fig. 102.

field of view of the observing telescope side by side and in the same plane. Let us assume now that both S_1 and S_2 are illuminated by monochromatic light, e.g., sodium light. The optical system can now be arranged so that the sodium-line image corresponding to S_1 coincides exactly with the sodium-line image from S_2. If this adjustment is made, then in any vertical line in the field of view visible radiation from the two sources illuminating S_1 and S_2 will be of the same wave length. By inserting a narrow vertical slit S_3 in the focal plane of L_3 any desired limited portion of the two spectra of the same wave length can be brought into

the field of view of an ocular system placed at such a distance to the rear of S_3 that S_3 lies in the focal plane of this ocular system.

The telescope tube T can be rotated through a small angle about an axis perpendicular to the plane of the paper by means of a graduated micrometer drum, which can be calibrated in terms of wave length. If T is rotated to such a position that the sodium light appears in the field of view between the vertical edges of the ocular slit S_3, this reading of the drum corresponds to a wave length of 5893 Ångströms. It is evident that the wave-length calibration curve of this instrument can be obtained readily by the use of standard spectral lines, such as those in the helium and mercury spectra.

Let us now assume that the instrument has been adjusted and calibrated for wave length. If the observer looks at the ocular slit S_3 without the aid of any ocular lens system, a "tri-partite" colored-light pattern as shown in Fig. 104 will be formed on the retina of the observer's eye. The dividing lines of this light pattern are horizontal. The central horizontal portion of this light pattern is due to light from S_2; the upper and lower horizontal portions are due to light from S_1. The intensity of illumination of the central portion can be varied by rotating the Nicol prism N_2. If the Nicol prism N_2 is set so that its angular scale reading is 90 degrees when the intensity of the light entering the observing tube T from S_2 is a maximum and equal to I_0, we may write

$$\frac{I_x}{I_0} = \sin^2 \theta \tag{104}$$

where θ is the angular setting of N_2 and I_x is the corresponding intensity of the light emerging from N_2. If θ is equal to zero, the intensity of the light emerging from N_2 is zero and the central portion of the "tri-partite" photometric light pattern is black.

Object:

The object of this experiment is to compare some source of illumination spectrophotometrically with some other source— that is, for several different wave lengths one source will be compared with a second source photometrically.

Place an "auxiliary" source of "white light" to be specified at S_2. Place one of the sources to be compared at S_1 and set the wave-length scale for any desired value. Rotate Nicol prism

N_2 so that equality of illumination in the "tri-partite" colored field is established, i.e., a photometric balance or "match" has been made so that the dividing lines in the "tri-partite" field have practically vanished. Read the angular setting of N_2 and call it θ_1. Now without disturbing the wave-length setting replace the source at S_1 by the source with which it is to be compared and proceed as before. Call the angular setting of N_2 in this case θ_2. For the particular wave length selected we may write

$$\frac{I_1}{I_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2} \tag{105}$$

where I_1/I_2 is the ratio of the intensity of the first source to the intensity of the second source.

Repeat the above procedure for several different wave lengths in the visible range, i.e., from about 4000 to about 7000 Ångströms.

Results Required:

Spectrophotometric curves of: (a) a "daylight" tungsten-filament incandescent electric lamp, and (b) a carbon-filament incandescent electric lamp, in terms of "daylight." These curves are plotted with the ratio I_1/I_2 on the y axis and wave length in μ or Ångströms on the x axis.

<div align="center">EXPERIMENT 77</div>

Spectrophotometric Measurements, Using as Spectrophotometer System the Nutting Polarization Photometer and the Constant-Deviation-Prism Spectrometer

Object:

The object of this experiment is to determine the per cent transmission of visible radiant energy through an absorption sample at several different wave lengths. The per cent transmission curve for the given sample co-ordinates wave length in the visible spectrum on the x axis and per cent transmission on the y axis.

Theory:

The optical system of the spectrophotometer to be employed is shown in Fig. 105 (a). A convenient source of illumination is placed at the focus S of the double collimator L_1L_2. Parallel light enters the Nutting polarization photometer at apertures

E_1 and E_2. The beam entering at E_1 passes through lens L_3 to the right-angle prism P_1, where it is reflected to the photometer cube P_2. The photometer cube P_2 consists of two right-angle prisms cemented together. A plane through YY perpendicular to the plane of the paper is the "plane of contact" of the two halves of the cube. The hypotenuse face of the left prism of cube P_2 is silvered with a narrow horizontal strip extending midway between the top and bottom of the cube. Some of the light which enters the cube P_2 after reflection from P_1 is reflected by the silvered strip so as to enter the "eye" lens L_E of the photometer, then passes through the rotating Nicol prism N_2 and enters "projec-
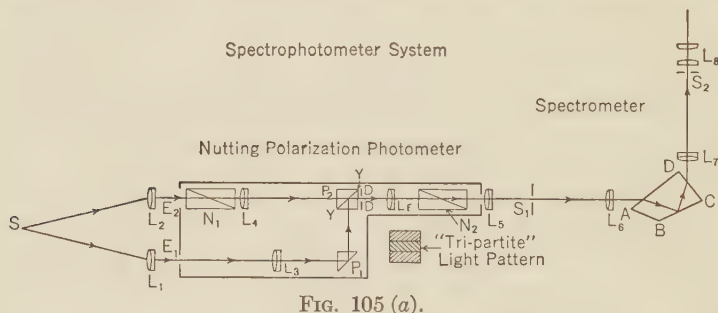
FIG. 105 (a).

tion" lens L_5. Lens L_3 brings the light entering at E_1 to a focus on the silver strip at the center of cube P_2. The projection lens L_5 is used to project an image of the light pattern at the center of the cube to the plane of the slit S_1 of the constant-deviation-prism spectrometer.

Parallel light entering at aperture E_2 passes through and is plane-polarized by the fixed Nicol prism N_1 and then passes through lens L_4. Some of this light then passes through cube P_2 over and under the horizontal silvered strip, enters the eye lens L_E, emerges from L_E, enters the rotating Nicol prism N_2, emerges from N_2 (if N_2 is not set for extinction), and enters the projection lens L_5. Lens L_4 brings the light entering at E_2 to a focus above and below the silvered strip in cube P_2. The projection lens L_5 projects this image to the plane of the slit S_1 of the spectrometer.

A diaphragm DD is placed immediately to the rear of P_2. There is a small rectangular aperture at the center of this diaphragm. This aperture acts as a necessary "field of view limiting stop."

It is now evident that the lens L_5 forms a "tri-partite" light pattern with dividing lines horizontal on the slit S_1 of the spectrometer. Slit S_1 and collimator lens L_6 form the collimator of the spectrometer; objective lens L_7 and ocular lens L_8 form the telescope of the spectrometer; $ABCD$ is the constant-deviation prism of the spectrometer. For any given setting of the wave length drum of the spectrometer there will appear in the field of view of the spectrometer telescope a "tri-partite" light pattern which consists of three sharply defined horizontal colored spectrum lines or "bands" arranged in a vertical line; the central band is due to light entering the Nutting photometer at E_1; the upper and lower bands are due to light entering at E_2. By inserting a narrow vertical slit S_2 in the focal plane of the ocular lens L_8 any desired limited portion of the two spectra of the same wave length can be brought into the field of view of the ocular system L_8. The intensity of illumination of the upper and lower portions of the "tri-partite" light pattern can be varied by rotating the Nicol prism N_2. If the Nicol prism N_2 is set so that its angular scale reading is 90 degrees when the intensity of the light over the path $E_2P_2L_EL_5$ is a maximum and equal to I_0, we may write

$$\frac{I_x}{I_0} = \sin^2 \theta \qquad (106)$$
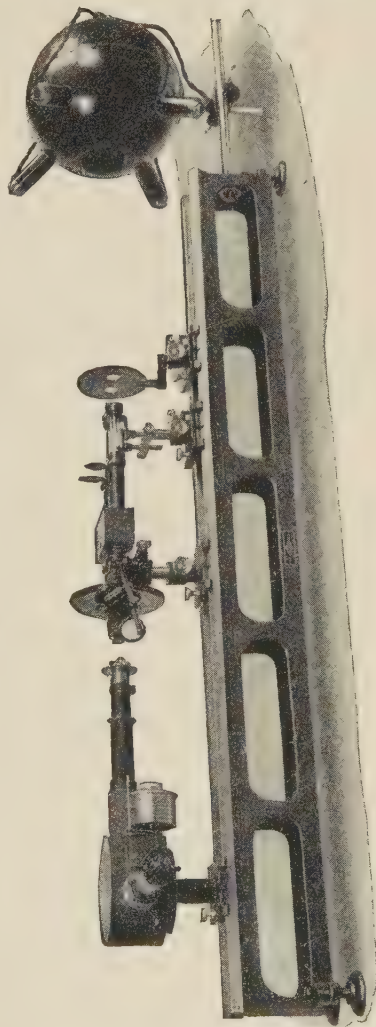
Fig. 105 (b).

where θ is the angular setting of N_2, and I_x is the corresponding intensity of the light emerging from N_2. If θ is equal to zero, the intensity of the light emerging from N_2 is zero, and the upper and lower portions of the "tri-partite" light pattern are black.

It is assumed that the student is familiar with the optical system and the manipulation of the constant-deviation-prism spectrometer. This instrument is discussed in Experiment 9.

Procedure:

Arrange the optical system as in Fig. 105 (a). With the specimen *not* in position between L_1 and E_1 and the wave-length drum of the spectrometer set to read any desired wave length, rotate Nicol prism N_2 so that equality of illumination in the "tri-partite" field is established, i.e., a photometric balance or "match" has been made so that the dividing lines in the "tri-partite" field have practically vanished. Read the angular setting of N_2 and call it θ_0. Now insert the test specimen between L_1 and E_1 and again read the angular setting of N_2 for which a photometric balance is established. Call this angle θ_x. The per cent transmission T of the test specimen for the wave length employed can be expressed as follows:

$$T = \frac{\sin^2 \theta_x}{\sin^2 \theta_0}(100) \tag{107}$$

Repeat the above procedure for several different wave lengths in the visible range, i.e., from about 4000 to about 7000 Ångströms. Figure 105 (b) shows the necessary instruments arranged in accordance with Fig. 105 (a).

Results Required:

Curve with per cent transmissions on the y axis and wave lengths in Ångströms on the x axis.

SECTION XVII

MISCELLANEOUS SUPPLEMENTARY EXPERIMENTS

EXPERIMENT 78

Study of the Optical System of the Human Eye by Use of the Kuehne Eye Model

Theory:

Figure 106 represents a horizontal section of the right eye of a human being, with first focal point F and second focal point F'. FF' is the optic axis. In the direction of the axis from the left
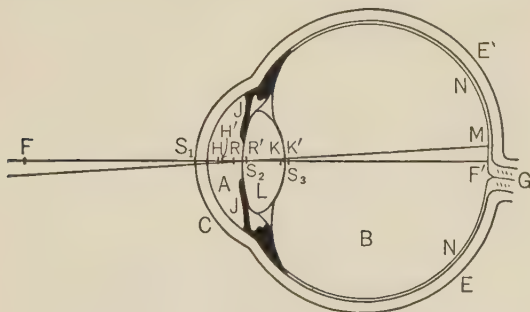
FIG. 106.

we have, first, the cornea C, which merges behind into the opaque sclerotic E surrounding the remainder of the eye. The latter is penetrated behind, somewhat to the nasal side, by the optic nerve, G. The thickness of the cornea is about 1 mm.; the radius of its inner (posterior) surface is smaller than that of the outer (anterior), so that the cornea, if it were surrounded by air, would act as a diverging lens. Behind the cornea is the anterior chamber A, filled with a watery fluid called the "aqueous humor" bounded behind by the iris J; this has a central circular aperture, the pupil, immediately behind which is the anterior surface of the eye-lens—called also the "crystalline lens"—composed of a series of fibers

arranged in layers. This lens is bi-convex, and when in a condition of rest, the anterior surface is considerably less curved than the posterior. Behind the crystalline lens is the large posterior chamber B, filled with a transparent gelatinous medium, called the "vitreous humor." This large chamber is surrounded, on the outside, by the outer layer, the sclerotic, within which is the choroid, of a dark color, and merging in front into the iris. Over the inside layer of the choroid spreads the retina, which is of complicated structure, and may be conceived as a continuation of the optic nerve. The retina is the screen, sensitive to light, on which the images of the external objects are formed, corresponding to the sensitized plate in the camera. At the point where the optic nerve enters the eye, the retina is not sensitive to light impressions; this spot is the blind spot (macula coeca). The part most sensitive to light is the yellow spot (macula lutea); it does not coincide with point F', where the optic axis cuts the retina, but lies towards the temporal side. The central depression or pit M of the yellow spot is called the fovea centralis, and is that point on which images are formed of objects that are under observation (fixed) by the eye.

On the axis are the cardinal points—focal points F and F', principal points H and H', nodal points K and K', and the centers R and R', of the entrance and exit pupils. The line drawn from M through R', which is continued through R and out into space, is called the line of vision or visual axis. It connects the point under observation (point of fixation) to the position of most distinct vision on the retina.

Table II gives the average constants of the human eye according to Helmholtz. An eye having the exact constants given in this table is known as a "schematic" eye. The first seven items in Table II have been obtained by measurement; the other values are calculated. The "positions" in items 6 and 7 and 10 to 17 inclusive are with reference to the anterior vertex of the cornea. The heading "Far" refers to the eye focused for infinity; heading "Near" refers to the eye accommodated to a point 152.5 mm. from the anterior vertex of the cornea.

Object:

The object of this experiment is to study the optical system of the human eye, and also some of the common defects and methods for correcting these defects. This study is to be made by the use

TABLE II

SCHEMATIC EYE—HELMHOLTZ

Distances in Millimeters

	Far	Near
1. Refractive index of aqueous and vitreous humor.	1.3365	1.3365
2. Refractive index of crystalline lens............	1.4371	1.4371
3. Radius of curvature of cornea................	7.829	7.829
4. Radius of anterior surface of crystalline lens....	10.0	6.0
5. Radius of posterior surface of crystalline lens...	− 6.0	− 5.5
6. Position of anterior surface of crystalline lens S_2.	3.6	3.2
7. Position of posterior surface of crystalline lens S_3.	7.2	7.2
8. First focal distance of eye....................	15.5	14.0
9. Second focal distance of eye.................	20.71	18.69
10. Position of first focal point...................	−13.74	−12.13
11. Position of second focal point................	22.82	20.95
12. Position of first principal point..............	1.75	1.86
13. Position of second principal point............	2.10	2.26
14. Position of first nodal point.................	6.97	6.57
15. Position of second nodal point...............	7.32	6.97
16. Position of center of entrance pupil...........	3.046	2.67
17. Position of center of exit pupil...............	3.705	3.298
18. Magnification in entrance pupil and exit pupil..	0.923	0.941

of the Kuehne eye model shown in Fig. 107. The Kuehne eye
model consists of a rectangular metal box open at the top and
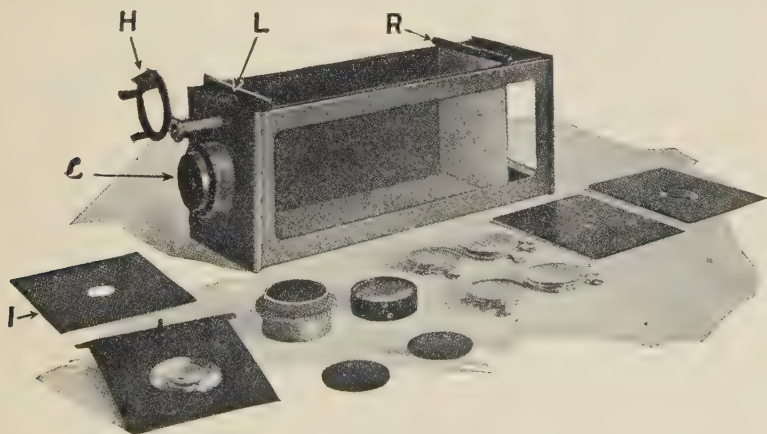
FIG. 107.

having one side and one end made of glass. A cornea C, an iris
diaphragm I, a "crystalline" lens L, and a piece of ground glass R

corresponding to the retina can be placed in their proper relative positions in the box. The whole box is filled with some liquid containing particles in suspension; a solution of eosin in water is satisfactory. The solution between the cornea and the crystalline lens corresponds to the aqueous humor, and the solution between the crystalline lens and the retina corresponds to the vitreous humor. In front of the cornea is a lens holder H for supporting ordinary "spectacle lenses." The complete equipment consists of the box, a normal cornea, an "astigmatized" cornea, an assortment of spherical and cylindrical "trial" spectacle lenses, several "crystalline" lenses, iris diaphragms of different sizes, ground-glass "retina," etc.

Procedure:

1. Place the normal cornea C_1, the iris diaphragm I_1, and the normal crystalline eye lens L_1 in the positions provided for them.
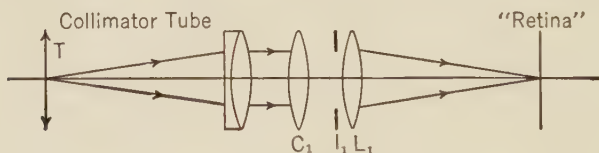
Fig. 108.

Fill the box with the eosin solution provided. Place the collimator tube in front of the cornea C_1, as shown in Fig. 108 and adjust the optical system so that the principal axes of the collimator and the eye-lens system are approximately coincident. The eye is now looking at a distant target. The small target T at the focus of the collimator will be illuminated by an electric lamp of high intensity so as to make the path of the light through the eosin solution easily visible.

Determine by aid of the graduated scale at the upper long edge of the box the "normal" position of the ground-glass "retina," i.e., the position for which a sharply defined image of target T is formed on the ground glass when the normal cornea C_1 and the normal crystalline lens L_1 are used.

2. Replace the normal crystalline lens L_1 by crystalline lens L_2. Determine the ocular defect and determine by use of the trial lenses supplied the dioptric power of the proper correcting lens.

3. Replace crystalline lens L_2 by crystalline lens L_3 and proceed as in step 2.

4. The instructor will now replace the normal cornea C_1 by cylindrical cornea C_2. Replace the normal crystalline lens L_1. Determine by means of the retina the position of: (a) the horizontal-line focus, (b) the vertical-line focus, (c) the circle of least confusion. Put the retina in the normal position determined under step 1. By use of the trial lenses supplied, determine the dioptric power and direction of the axis of the proper cylindrical correcting lens.

5. Replace the cylindrical cornea C_2 by the normal cornea C_1 and remove the crystalline lens employed in step 4. Try to determine a position of the retina for which a sharply defined image of T is formed by means of the normal cornea. Put the retina in the normal position and by use of the trial lenses determine the dioptric power of the proper correcting lens.

6. Using the normal cornea and normal crystalline lens place the retina in the normal position. Test the eye for longitudinal spherical aberration by using iris diaphragms of different sizes.

7. Remove the collimator system. Place the illuminated target supplied at a distance to be specified by the instructor. Using the normal cornea place the retina in the normal position and determine by trial the particular crystalline lens which will form a sharp image of the target on the retina. Repeat this procedure for two other positions of the target. These distances will also be specified by the instructor.

Results Required:

Statement of ocular defect in steps 2, 3, and 4 above, and a written "prescription" for each case.

Conclusions from step 5.

Conclusions from step 6.

Conclusion regarding "accommodation" from step 7.

Acknowledgment:

Figure 106 and the first three paragraphs of this experiment are taken from Gleichen's "Theory of Modern Optical Instruments."

Interference Method for Determining Chromatic or Color Errors in a Lens

Method:

In Fig. 109(a), S is a vertical slit illuminated by some form of monochromatic illuminator. S is placed at the focus of a well-
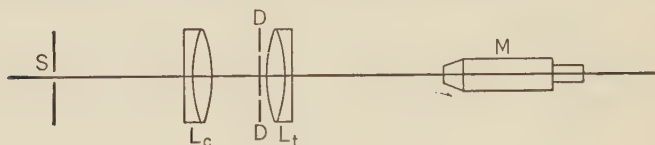
Fig. 109 (a).

corrected collimator lens L_c. L_t is the test lens, M is a measuring microscope, and DD is a diaphragm having two vertical slits as
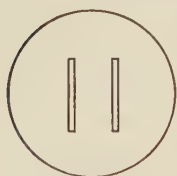
Fig. 109 (b).

shown in Fig. 109 (b). Colored interference fringes will appear in the focal plane of L_t. These fringes can be counted and measured by means of M. If "color magnification error" is present the fringe width will vary with the wave length of the light illuminating S. If "color position error," i.e., axial chromatism, is present, M must be shifted as the wave length changes, assuming L_t not disturbed.

Results Required:

"Color magnification error" as per cent difference between C and F light. "Color position error" as the ratio of $(f_c - f_F)$ to f_D.

Measurement of Chromatic or Color Errors in a Telescope

Method:

1. If the ratio of the diameter of the entrance pupil to the diameter of the exit pupil is the same for all wave lengths, the magnification of the telescope is independent of wave length, and consequently there is no "magnification color error" present.

2. Assume a telescope focused on a fairly distant vertical slit illuminated by white light, the telescope being so pointed that the

image of the slit appears near the edge of the field.   If a color filter which transmits, say the C and F lines, is placed between the ocular and the eye of the observer, the red and blue images of the slit will be seen displaced slightly at the edge of the field if "color magnification error" is present.   If the slit can be illuminated first by C light and then by F light, or by C and F light simultaneously, a color filter will not be necessary to carry out this idea.

3. Assume the telescope in step 2 so pointed that the image of the slit is at the center of the field.   If the C and F images are not in focus simultaneously, there is "color position error" present, that is, axial chromatism.

Results Required:

"Color magnification error" under steps 1 and 2 expressed in angular measure.   "Color position error" under step 3 in millimeters.

<div align="center">EXPERIMENT 81</div>

Measurement of Distortion in a Telescope Due to the Ocular

Method:

Assume the magnifying power of the telescope determined by two methods as follows:

(a) Measure the diameter of the entrance pupil and the diameter of the exit pupil.   Call the ratio M_1 of the former to the latter the magnifying power.

(b) Measure the "apparent field" ω_A and the true field ω_t. The magnifying power M_2 is

$$M_2 = \frac{\tan \dfrac{\omega_A}{2}}{\tan \dfrac{\omega_t}{2}} \tag{108}$$

Very often M_1 and M_2 are found to be unequal.   This disagreement between M_1 and M_2 can usually be traced to the ocular, and it is due to the fact that the magnification produced by the ocular is different in different positions of the field, that is, there is distortion present.

An optical arrangement which can be employed to test the ocular for distortion is shown in Fig. 110. $S_1 S_2$ is a linear scale,

L_p is a "projection" lens, $S'_1 S'_2$ is an undistorted image of $S_1 S_2$ formed by L_p, E_x is the ocular being tested. E_x is so placed with respect to $S'_1 S'_2$ that an enlarged image of $S'_1 S'_2$ is formed on the screen XY. If there is no distortion present in E_x the ratio of the size of the image of any portion of $S'_1 S'_2$ to the corresponding portion of $S'_1 S'_2$ itself will be the same regardless of what portion of $S'_1 S'_2$ is being considered. Let us assume that 5 mm. in $S'_1 S'_2$ correspond to 96 mm. in the enlarged image formed on screen XY and that 10 mm. in $S'_1 S'_2$ correspond to 200 mm. in the enlarged image formed on screen XY. These figures indicate that the
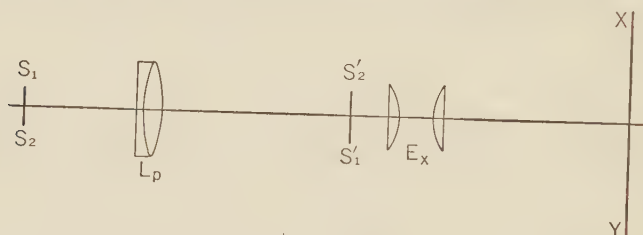
Fig. 110.

ocular E_x produces different magnifications at different parts of the field, that is, there is distortion present. For the general case the image size y, the object size x, and the magnification a can be connection by the relation

$$y = ax + bx^2 \qquad (109)$$

where b is the "distortion coefficient." Applying equation (109) to the data assumed above we obtain results as follows:

$$a = 18.4$$
$$b = 0.16$$

Hence equation (109) may now be written

$$y = 18.4x + 0.16x^2 \qquad (110)$$

If an ocular which performs according to equation (109) is used in a telescope, it is obvious that the magnifications M_1 and M_2 mentioned above will not be equal. For a certain standard telescope the measured value of $M_1 = 6.6$ and the measured value

of $M_2 = 7.2$. The magnification in such a case can be expressed as follows:

$$M_2 = M_1 + k\omega^2 \tag{111}$$

where ω is the angular field of view being observed, and k is a constant.

In the case of the standard telescope mentioned above it was found that equation (111) reduced to

$$M_2 = 6.6 + 0.044\omega^2 \tag{112}$$

Procedure:

1. For the telescope supplied measure M_1 and M_2 as indicated in (a) and (b) under "method." ω_t as determined in (b) is the maximum value of ω in equation (111). Substitute these measured values of M_1, M_2, and ω in equation (111) and solve for k.

2. Remove the ocular and arrange the optical system as in Fig. 110. Obtain sufficient data so that the constants a and b in equation (109) can be determined.

Results Required:

Show the connection between the results obtained under steps 1 and 2 of "Procedure."

EXPERIMENT 82

Testing a Telescope for Coma

Procedure:

1. Focus the telescope on a fairly distant test star, i.e., a very small illuminated pinhole. If the best images obtainable at points removed from the center of the field have "tails," i.e., the images suggest comets in appearance rather than points, there is coma present.

2. Focus the telescope on an illuminated target which consists of a piece of ground glass on which a vertical black bar appears. Point the telescope so that the image of the bar appears at the edge of the field. If coma is present, the image of the black bar will not look perfectly black, but will be hazy, that is, it will look as though an appreciable amount of light were "creeping" from the white or translucent portion of the target into the black bar; if no coma is present, this hazy effect does not appear and the bar image is very black.

<div align="center">

EXPERIMENT 83

Determination of Aberrations of a Lens
by Means of the Interferometer

</div>

Theory:

In Fig. 111, S is a point source of monochromatic light placed at the focus of collimator lens L_c; P is a plane parallel glass plate half-silvered on the surface facing the test lens L_x; M_c is a convex spherical full-silvered mirror with center of curvature at C; M_1 is a plane full-silvered mirror; and L_0 is a "collector" lens having its focus at F. Part of the light arriving at the half-silvered sur-

<div align="center">
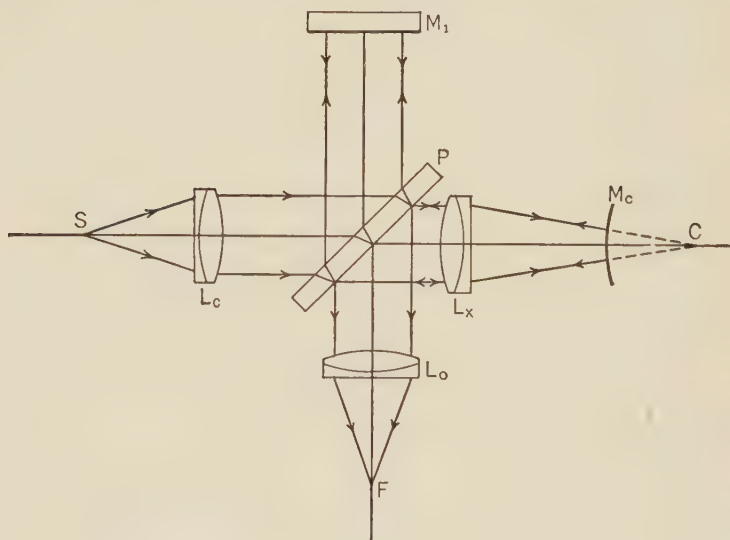
Fig. 111.

</div>

face of P is reflected to mirror M_1, where it is again reflected so as to pass through P and L_0 and to form an image of S at F. The other part of the light arriving at the half-silvered surface of P is transmitted through L_x, is reflected by convex mirror M_c, passes back through L_x to the half-silvered surface of P, where it is reflected and passes through L_0 to form an image of S at F. The two beams arriving at F are in condition to interfere and when the proper adjustments have been made interference bands can be seen by an eye placed to the rear of F. If the adjustments have been properly made, the test lens L_x, if aberrationless, will

receive the beam of plane wave front and return it with a plane wave front; if it does not do so, aberrations are present, and the departures from planeness of the returned wave front will form a contour map of the corrections which must be applied to the lens in order to make it aberrationless.

Figure 112 shows the appearance of the interference bands corresponding to fairly pure examples of the five aberrations: spherical aberration, coma, astigmatism, curvature of field, and distortion, respectively.

SPHERICAL ABERRATION   COMA   ASTIGMATISM   CURVATURE OF FIELD   DISTORTION

FIG. 112.

Results Required:

Contour map of the interference fringes for each test lens supplied and conclusions.

Note.—Lord Rayleigh is responsible for an important rule, which he expressed as follows:

An obvious inference from the necessary imperfection of optical images is the uselessness of attempting anything like an absolute destruction of aberration. In an instrument free from aberration the waves arrive at the focal point in the same phase. It will suffice for practical purposes if the error of phase nowhere exceeds $\lambda/4$. This corresponds to an error of $\lambda/8$ in a reflecting and $\lambda/2$ in a (glass) refracting surface, the incidence in both cases being perpendicular.

It is found by experience that the quarter wave-length rule just stated is a reliable one. The departure (expressed in wave lengths) from a plane or spherical wave surface of the wave surface as it leaves a lens forms the basis of measurements of imperfections in definition of the image formed by such a lens. In other words, the aberrations as expressed in geometrical optics can be determined from the indications of a specially arranged interferometer. Such an interferometer is built by Adam Hilger, Ltd., London, England.

The method employed in this experiment is due to F. Twyman of the Research Department of Adam Hilger, Ltd., London, England.

EXPERIMENT 84

Determination of Two-Constant Dispersion Formulae
Due to Cauchy and to Nutting

Discussion:

Considerable time is involved in obtaining a complete dispersion curve for a sample of optical glass. Where an extremely high degree of accuracy is not required, the index of refraction n can be determined as a function of the wave length λ from so-called two-constant dispersion formulae. Two such dispersion formulae will be considered. The first is due to Cauchy and reads as follows:

$$n = a + \frac{b}{\lambda^2} \tag{113}$$

or

$$n - 1 = a' + \frac{b}{\lambda^2} \tag{114}$$

where a, a', and b are constants.

The second is due to Nutting and reads as follows:

$$\frac{1}{n-1} = c + \frac{d}{\lambda^2} \tag{115}$$

where c and d are constants.

The great utility of these dispersion formulae lies in the fact that n corresponding to any wave length λ can be very quickly calculated as soon as the two constants have been determined.

Procedure:

By the method of Experiment 4 obtain a sufficient number of simultaneous pairs of values of n and λ to permit an accurate determination of the constants in equations (113), (114), and (115).

EXPERIMENT 85

Qualitative Determination of Aberrations of a Lens
by " Test-Chart " Method

Method:

The method in this case consists in forming by means of the test lens an image of a test chart of the general form shown in Fig.

113.   The design of this chart is such as to bring out those aberrations present in the test lens.

The image should be inspected with the aid of a magnifying glass for the following:

1. Definition.
2. Coma.
3. Astigmatism.
4. Distortion.
5. Curvature of field.

If the edges of the heavy black bars do not appear sharp, that is, good definition is lacking, chromatic or spherical aberration,
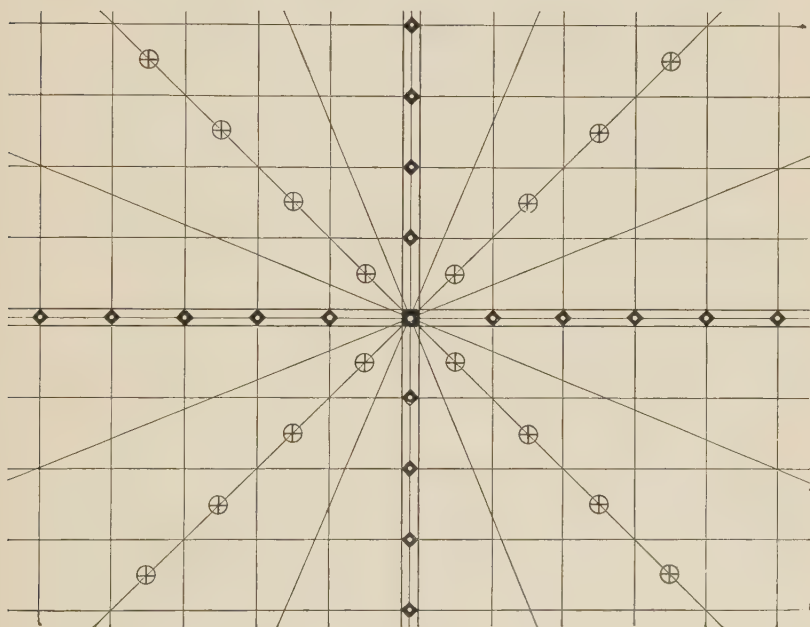
Fig. 113.

or both, are present.   Place a color filter in front of the lens and inspect the image again for definition.   If the definition is better than before, chromatic aberration is present.   With the color filter still in position "stop down" the lens by means of a diaphragm with a central opening and again inspect the image for

definition. If the definition is improved, spherical aberration is present.

If the small white circles inside the heavy black bars are blurred or slightly "comet shaped," or if the ends of the black bars are hazy, i.e., light has crept into them, coma is present. If certain of the inclined lines lack sharpness while those at right angles are sharply defined, astigmatism is present.

If the outer lines of the image are slightly curved, or if the magnification is not the same at the edges as at the center, distortion is present.

Assume that the definition is very good at the center but not at the edges. By changing the distance between the lens and the image the definition at the center of course will be impaired, but the definition at the edges may improve. In such a case the presence of curvature of field is obvious.

This method can, of course, be carried out photographically. The focus should be adjusted with the aid of a magnifying glass before the exposure is made and the inspection of the finished negative should be made with the aid of a magnifying glass.

It is advisable, whenever possible, to combine this "chart" test with some of the other tests for aberrations described in this manual

Procedure:

Obvious from discussion of "Method."

Results Required:

Report of findings.

Acknowledgment:

The general plan of this test is taken from Johnson's "Practical Optics," published by Benn Brothers, Ltd., London.

<div align="center">EXPERIMENT 86</div>

<div align="center">Determination of Per Cent Light Transmission for Any Telescopic
Instrument by Use of a Modification of the
Macbeth Illuminometer</div>

Definition:

Per cent light transmission for any telescopic instrument is defined as the ratio, expressed in per cent, of the brightness of the emergent light to the brightness of the incident light.

Method:

The optical arrangement for this method is shown in Fig. 114. DS is a uniformly illuminated white diffusing surface placed at the focus of the collimator lens L_c. L_0 and L_E are the objective and ocular lenses, respectively, of the test telescope. The reader is advised at this point to refer to Experiment 42 and to read carefully the description of the Macbeth illuminometer. For measuring per cent light transmission through a telescope the Macbeth illuminometer is modified as follows:

1. A lens of short focal length is placed in a draw-tube and this draw-tube is pushed into the sighting aperture S, Fig. 73.
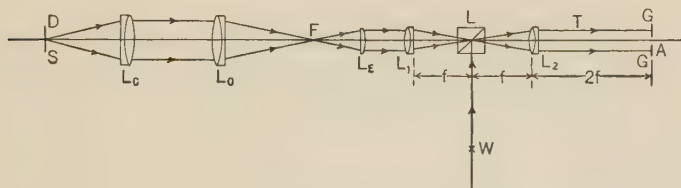
Fig. 114.

This lens L_1, Fig. 114, is placed so that the center of the photometer cube L is at the right focus of the lens.

2. The tube T, Fig. 73, is removed and is replaced by a tube containing a lens L_2, Fig. 114, which is an exact duplicate of lens L_1. L_2 is placed so that the center of the photometer cube L is at the left focus of L_2.

Lenses L_1 and L_2 form a telescope of unit magnification. This telescope will be called the photometer telescope. L_1 and L_2 are separated by a distance $2f$, and the exit pupil of this telescope is at a distance $2f$ to the right of L_2, where f is the focal length of the similar lenses L_1 and L_2. GG is a circular piece of ground glass placed in the plane of the exit pupil of the photometer telescope. At the center of GG is a small circular aperture A of diameter 2 mm., which is the minimum diameter of the pupil of the human eye. GG is mounted in the tube which contains L_2. W is the working standard of the Macbeth illuminometer.

Lens L_0 forms an image of DS at F and L_1 forms a second image of DS at the center of cube L. The path of the rays from the center of DS is shown in Fig. 114.

Procedure:

With the test telescope *not* in position adjust the size of a circular aperture of variable opening placed in front of DS so that the size of the exit pupil as observed on GG is slightly greater than the diameter of A. Move W to a position such that a photometric balance is obtained. Read the scale of the instrument and call it S_1. Now place the test telescope between L_c and L_1 as shown in Fig. 114 and again obtain a photometric balance. Read the scale for this case and call it S_2. Keeping in mind the fact that the Macbeth illuminometer scale is an "inverse-square" scale, it is obvious that the per cent light transmission t can be expressed as follows:

$$t = \frac{S_2}{S_1}(100) \tag{116}$$

Results Required:

Per cent light transmission for each telescopic instrument supplied.

Acknowledgment:

This method for measuring per cent light transmission through a telescopic instrument was called to the attention of the author by A. H. Bennett, Associate Physicist, Bureau of Standards, Washington, D. C.

EXPERIMENT 87

Determination of Per Cent Light Transmission for any Telescopic Instrument by Use of the Nutting Polarization Photometer

Method:

The method in this experiment is to be worked out by the student after reading carefully the description of the Nutting polarization photometer in Experiment 43 and the discussion of the "Method" in Experiment 86.

Results Required:

Per cent light transmission for each telescopic instrument supplied.

EXPERIMENT 88

Detection of Striae in Optical Glass

Method:

The method to be employed in this experiment is taken from Bureau of Standards Scientific Paper No. 373, by Smith, Bennett, and Merritt. The reader is advised to read this paper carefully, particularly with reference to the subject of characteristics of striae in optical glass.

In this method the striae are projected optically either upon a suitable screen or upon a photographic plate if a permanent record is desired.

Procedure for Slabs and Prisms:

Arrange the optical system as in Fig. 115. A is a "point source" of high intensity (tungsten arc or its equivalent), L_c is
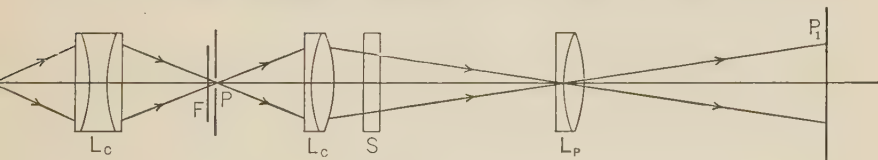
FIG. 115.

a condenser lens system, F is a filter, P is a pinhole, L_c is a collector lens, S is the test slab of optical glass, L_p is a projection lens, and P_1 is the screen or photographic plate. If the test slab is placed so that its image is in sharp focus on P_1, the striae are faint; if the image is thrown slightly out of focus, the striae show up clearly. A stria which appears dark with P_1 on one side of the sharp-focus position will appear bright when P_1 is moved to the other side of the focus. This fact makes it possible to distinguish between a scratch or bubble and a stria.

The arrangement just discussed for projecting striae in a slab of optical glass can also be used for prisms.

Procedure for Lenses:

Fairly heavy striae in a lens can be detected by observing their shadows formed on a diffusing screen placed between the lens and the focus of an approximate point source, such as the sun.

Arrange the optical system as in Fig. 116. H is an illuminated pinhole, L_T is the test lens, L_p is a projection lens, and P is the screen or photographic plate.

Results Required:

Sketches or photographs of the striae and conclusions.
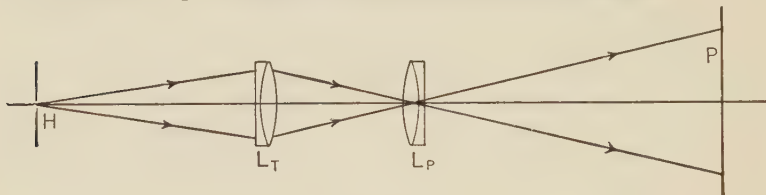
FIG. 116.

EXPERIMENT 89

Determination of Focal Length of a Simple Diverging Lens by an Approximate Method

Method:

In Fig. 117, L_x is the diverging test lens with its left principal focus at F, and RR' is an opaque diaphragm having a circular
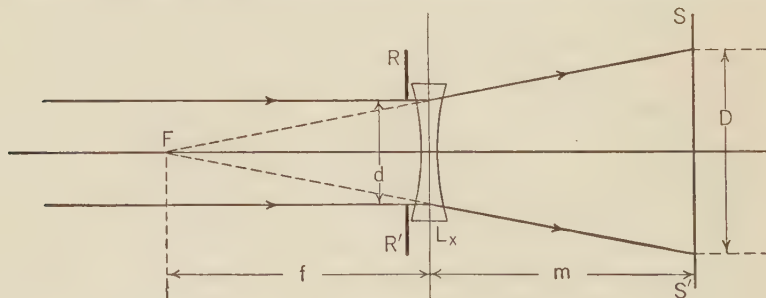
FIG. 117.

aperture of diameter d. An intense beam of rays parallel to the principal axis either from the sun or from a collimator is incident on L_x. The diameter of the beam is d. The rays emerge from L_x as a divergent beam and form a circular-light pattern on screen SS'. If screen SS' is moved to a position such that the diameter D of this light pattern is equal to $2d$, it is obvious that the distance m between L_x and SS' is the approximate focal length f of the test lens L_x.

Procedure:

Obvious from discussion of "Method."

Results Required:

Focal lengths of diverging lenses supplied.

Question:

Why is this method approximate?

EXPERIMENT 90

Measurement of Curvature of Field and Astigmatism of a Telescope

Procedure:

1. Focus the telescope on a distant target graduated in degrees or on its optical equivalent, the Angular Scale Target described in Part 1 of the Miscellaneous Introductory Notes. Adjust the focus so that the best possible definition is obtained at the center of the field. If the field is flat the outer portions of the target image will also be in good focus; if the field is not flat portions of the target image other than the central portion will be out of focus.

2. In order to increase the accuracy of the observations and also *to eliminate the accommodation of the observer's eye*, place between the eye and the telescope under test a small auxiliary telescope having a magnification of about three, that is, look into the ocular of the test telescope with the aid of the auxiliary telescope in steps 3 and 4 below. The auxiliary telescope should be focused for infinity.

3. Align the two telescopes so that the zero of the target image is at the center of the field of view. Note the position of the ocular of the test telescope on the axis of the telescope. Now adjust the ocular of the test telescope so as to give the best possible definition for the one degree circle. Note the position of the test telescope ocular on the telescope axis when the horizontal part of the one degree intersection is in focus and also when the vertical part is in focus; the first position of the ocular locates the secondary image plane and the second position locates the primary image plane. In this manner we can determine any lack of flatness of the image surface and we have data for calculating the curvature of field and astigmatism for points one degree from the axis.

4. Repeat step 3 for the two degree circle, the three degree circle, etc., to the extreme edge of the field.

Interpretation of Data:

In order to interpret the data obtained it is necessary to express the results in *displacements per diopter*. It can easily be shown that the number of millimeters displacement per diopter, ΔX, can be expressed as follows:

$$\Delta X = \frac{F^2}{1000} \tag{117}$$

where F is the focal length of the ocular of the test telescope in millimeters.

In order to illustrate the use of equation (117) assume two oculars designed exactly alike, one having a focal length of 40 mm. and the other 10 mm. Also assume that the curvature of field at $20°$ is 10 per cent of the focal length. In order to focus the 40 mm. ocular the displacement is 4 mm. and from equation (117) this represents 2.5 diopters. In order to focus the 10 mm. ocular the displacement is 1.0 mm., and this represents 10 diopters. The field in the case of the 40 mm. ocular will appear flat because the eye is called upon to accommodate through a range of only 2.5 diopters. Good definition will be lacking when the 10 mm. ocular is used because the eye does not have an accommodation range of 10 diopters.

Results Required:

Curvature of field for secondary and primary focal lines as per cent of effective focal length of ocular, and astigmatic differences for oblique rays incident at angular values ranging from zero to full field of the telescope. Curves similar to those shown in Fig. 61 to show departure from a flat field.

Note:

The student should read carefully the method of Experiment 33 before performing this experiment.

APPENDIX I

USEFUL LABORATORY INFORMATION

1. SILVERING OPTICAL SURFACES

Prepare two solutions A and B as follows:

A

Silver nitrate............................... .	5 grams
Distilled water................................	40 c.c.

To this add ammonia slowly until the precipitate which is at first formed is nearly redissolved. The success of the solution depends upon leaving an excess of the precipitate. If a drop too much ammonia has been added, a small crystal of silver nitrate must be put in to bring back traces of the precipitate. When the solution is right it will look like slightly muddy water. Then dilute to 500 c.c. and filter.

B

Silver nitrate................................	1 gram
Rochelle salt (sodium-potassium tartrate).........	0.83 gram
Distilled water................................	500 c.c.

Bring the solution to a boil and filter hot. It must be cooled to the temperature of the room before it is used.

For silvering use equal parts of A and B.

The essential of success in silvering is, besides the ammonia in solution A, cleanliness. The following process of cleaning the surfaces to be silvered is recommended:

1. Remove wax, if there is any, with spirits of turpentine.

2. Wash off the turpentine with soap and water.

3. Place the surfaces to be silvered in a glass or porcelain tray, and remove any remaining silver with strong nitric acid.

4. Rinse well in running water.

5. Wash in a strong solution of caustic potash. During this

washing the plates and the dish which holds them should be rubbed hard with a tuft of cotton or a piece of pure gum tubing on the end of a glass rod. The success of the process depends largely upon the thoroughness of this washing.

6. Pour off the solution of caustic potash and rinse well in running water, being careful not to touch either the surfaces or the inside of the dish with the fingers. A very minute trace of grease will make the film streaked.

7. Wash in strong nitric acid.

8. Wash in running water for five minutes or more, raising the surfaces with a glass rod to allow the water to run beneath them.

9. Rinse in several changes of distilled water.

Now mix the two solutions and pour over the surfaces. If only a thin coat is desired, the deposit must be watched, and the surface removed when the film has the necessary thickness. The opaque films should remain in the solution till it turns black. The surfaces are then removed and set up on edge on filter paper to dry. When dry they may be polished by rubbing them gently on a piece of chamois skin laid on the table and covered with jewelers' rouge. The transparent films cannot be polished, for a slight touch will rub the coating off entirely. If the solution is successful, the opaque films will be so hard that they cannot be rubbed off with the finger.

2. PREPARATION OF A "HALF-SILVERED" MIRROR OR TRANSPARENT FILM OF SILVER ON GLASS

(From Research Department, Adam Hilger, Ltd., London, England.)

Prepare four solutions, A, B, C, and D as follows:

A

10 per cent silver nitrate.

B

40 per cent formalin.

C

Granulated sugar	400 grams
Ethyl alcohol	200 c.c.
Concentrated nitric acid	10 c.c.

Add 2000 c.c. distilled water and allow to stand for two weeks before using.

D

Chromic acid.................................... 250 grams
Concentrated sulphuric acid..................... 1500 c.c.

Procedure:

Place the glass plate to be half-silvered in a clean glass vessel and clean it thoroughly by "swabbing" with concentrated nitric acid. Now place the glass plate in a sufficient amount of solution D for about 30 seconds. Pour off solution D and rinse the plate and glass container thoroughly with running water to remove every trace of solution D. Take 20 c.c. of solution A, add ammonia until the precipitate is *just* redissolved, and add silver nitrate solution (any strength) until the solution is a faint straw color. Add distilled water to make the volume 100 c.c. Place the glass plate face up or down in this ammoniacal silver solution. Now prepare a "reducing solution" as follows:

$$\left.\begin{array}{l} \text{5 c.c. of B} \\ \text{5 c.c. of C} \end{array}\right\} \text{mixed}$$

Now add the reducing solution. Agitate the mixture until the the solution becomes reddish in color. Pour off the solution and replace it by a second quantity of the ammoniacal silver solution, *adding no reducing solution*. Allow the "mirror" to remain until the desired thickness of deposit is obtained—a few minutes will be sufficient. Now rinse the "mirror" thoroughly with distilled water and dry it. Polishing is not necessary.

3. "FROSTING" SOLUTION FOR GLASS OR "GROUND-GLASS SUBSTITUTE"

(From Johnson's " Practical Optics.)

A satisfactory "frosting solution" can be prepared as follows:
Dissolve 100 grams of gelatin (leaf) and 480 grams of either calcium carbonate or magnesium oxide in 1 liter of hot distilled water.

Allow the solution to cool to 34 degrees C. and dip the glass into it. When dry immerse the glass a second time. Add more "coats" if necessary.

A very satisfactory "ground-glass substitute" or "frosting solution" can be purchased from the Eastman Kodak Company.

4. FLUORESCENT SCREENS

(For Observatious in the Ultra-Violet)

Moisten a small quantity of anthracene with water and apply a thin layer over a piece of ground glass. When dry, most of the anthracene will adhere to the glass. Place the surface of the glass on which the anthracene is deposited so as to receive the radiation directly. If the glass is turned the other way most of the ultra-violet radiation will be absorbed by the glass and consequently will not reach the anthracene.

5. LABORATORY CEMENT

For cementing glass to metal, glass to glass, etc., for experimental work in the laboratory, De Khotinsky cement or "Kotinsky" is recommended. This cement can usually be purchased from dealers in physics laboratory equipment and supplies.

A satisfactory cement for the purposes just mentioned can be prepared by mixing equal masses of beeswax and rosin while molten. On cooling it can be made into "sticks" of convenient size.

6. UNIVERSAL SOFT WAX

Melt together 1 part by weight of Venice turpentine to 5 parts by weight of beeswax. Color with vermilion if desired.

APPENDIX II

PHOTOGRAPHIC FORMULAE

The photographic formulae given below are the standard formulae published by the Eastman Kodak Company. While particularly adapted for use in connection with the photographic emulsions manufactured by this company, these formulae can usually be used successfully for most laboratory purposes with photographic emulsions manufactured by other companies.

1. STANDARD A.B.C. PYRO (D–1) FOR NEGATIVE MATERIAL ONLY

Stock Solution A

Sodium bisulphite or potassium meta-bisulphite.....................	140 grains	9 grams
Pyro...............................	2 oz.	60 grams
Potassium bromide.................	16 grains	1 gram
Water to make.....................	32 oz.	1000 c.c.

Stock Solution B

Water...........................	32 oz.	1000 c.c.
Sodium sulphite....................	3.5 oz.	105 grams

Stock Solution C

Water...........................	32 oz.	1000 c.c.
Sodium carbonate..................	2.5 oz.	75 grams

For Tray Development

Take 1 volume of A, 1 volume of B, 1 volume of C, and 7 volumes of water.

2. PYRO ELON STOCK SOLUTION FOR NEGATIVES ONLY

A

Water..	500 c.c.
Sodium bisulphite or potassium metabisulphite......	2 grams
Potassium bromide.............................	1 gram
Elon..	4 grams
Pyro..	15 grams

B

Water..	500 c.c.
Sodium sulphite................................	90 grams

C

Water..	500 c.c.
Sodium carbonate..............................	90 grams

For Tray Development

Use 1 volume of A, 1 of B, 1 of C, and 10 of water for developing ordinary material.

This Pyro developer does not stain the fingers as much as the previous one.

3. ELON HYDROCHINON DEVELOPER STOCK SOLUTION FOR NEGATIVES

A

Water..	2000 c.c.
Elon..	8 grams
Sodium sulphite................................	60 grams
Hydrochinon...................................	8 grams
Potassium bromide.............................	4 grams

B

Water..	500 c.c.
Sodium carbonate..............................	45 grams

To develop ordinary negative material in trays use 4 volumes of A, 1 volume of B, 4 volumes of water.

4. CONTRAST DEVELOPER FOR NEGATIVES (D–11)

Hot water (50° C.)............................... 2000 c.c.
Elon.. 4 grams
Sodium sulphite................................. 300 grams
Hydrochinon..................................... 37.5 grams
Potassium carbonate............................. 97.5 grams
Potassium bromide............................... 22.5 grams
Water to make................................... 4000 c.c.

For Tray Development

Develop about 5 minutes at 65° F. If less density is desired, dilute with an equal volume of water.

Where a high degree of contrast is desired use this developer with "Process" negative material. "Process" emulsions give high contrast values.

5. PRINT DEVELOPER STOCK SOLUTION (CONCENTRATED)

Dissolve in 600 c.c. of hot water (50° C.) in order named:

Elon.. 4 grams
Sodium sulphite................................. 60 grams
Hydrochinon..................................... 15 grams
Sodium carbonate................................ 45 grams
Potassium bromide............................... 2 grams
Wood alcohol.................................... 90 c.c.

For Azo use 1 volume of stock to 7 of water and 2 drops saturated KBr solution.

For bromide paper use 1 volume of stock to 6 of water.

6. PRINT DEVELOPER STOCK SOLUTION (D–67).

For Velox, Azo, bromide paper, and lantern slides.

Hot water (about 50° C.)........................ 500 c.c.
Elon.. 3 grams
Sodium sulphite................................. 45 grams
Hydrochinon..................................... 9 grams
Sodium carbonate................................ 75 grams
Potassium bromide............................... 1 gram
Cold water to make.............................. 1000 c.c.

For Velox dilute 1 to 1.

For Azo and lantern slides dilute 1 to 2.

For bromide paper and softer results on lantern slides dilute 1 to 4.

7. LIQUID HARDENER

Dissolve in order given:

Water..	1000 c.c.
Sodium sulphite................................	200 grams
Acetic acid (25 per cent solution)................	600 c.c.
Alum..	200 grams

Use 1 volume of liquid hardener to 8 volumes of plain hypo solution. "Plain hypo solution" consists of about 1 volume of granular hypo dissolved in 4 volumes of water. Specific gravity of plain hypo solution should be about 1.110 at 20° C.

The use of liquid hardener in a hypo solution is always to be recommended for fixing prints. While not so necessary in the case of negative materials, it is also recommended in this case.

8. "SHORT STOP" SOLUTION

1 volume of a 25 per cent solution of acetic acid to 20 volumes of water.

"Short stop" solution is used to arrest the development of prints. The print is immersed for a few seconds in this solution as soon as it leaves the developing solution and is then put into the fixing solution.

9. PLAIN HYPO STOCK SOLUTION

Add granular hypo to water so as to make the specific gravity of the solution 1.110 at about 20° C.

10. NEGATIVE RINSE BATH

Water............................	1 gal.	4000 c.c.
Potassium chrome alum..............	2 oz.	60 grams
Acetic acid (28 per cent).............	3 oz.	90 c.c.

The negative material is immersed for a few seconds in this solution as soon as it leaves the developing solution and is then put into the fixing solution.

11. TRAY-CLEANING SOLUTION

Water.. 1000 c.c.
Sodium bichromate..............................   90 grams
H_2SO_4 (concentrated)............................   90 grams

This cleaning solution is particularly recommended for white enameled trays. Clean the tray as soon as possible and do not keep the tray-cleaning solution in the tray more than a minute or two.

APPENDIX III

OPTICAL AND MATHEMATICAL TABLES

TABLE I

WAVE LENGTHS IN ÅNGSTRÖMS CORRESPONDING TO THE
FRAUNHOFER LINES

Lines	Wave Lengths in Ångströms
A	7594
B	6867
C	6563
D_1	5896
D_2	5890
E	5270
F	4861
G'	4341
G	4308
H	3968
K	3934

TABLE II

APPROXIMATE INDEX OF REFRACTION OF COMMON SUBSTANCES
FOR THE D LINE $(\lambda = 5893$ Ångströms$)$

Substance	Index of Refraction
Air	1.00
Canada balsam	1.54
Carbon bisulphide	1.618
Ethyl alcohol	1.361
Water	1.333
Calcspar (ordinary ray)	1.658
Calcspar (extraordinary ray)	1.486
Diamond	2.43
Glass	1.5–2.0
Rock salt	1.544
Quartz (ordinary ray)	1.544
Quartz (extraordinary ray)	1.553

TABLE III

Wave Lengths Commonly Used in the Laboratory
(Visible Spectrum Only)

Element	How Produced	Wave Length in Ångströms	"Approximate Color"
Cadmium.........	Vacuum tube	6439	Red
		5086	Green
		4800	Blue
Helium............	Vacuum tube	7065	Red
		6678	Red
		5876	Yellow
		5048	Green
		5016	Green
		4922	Green
		4713	Blue
		4471	Blue
		4389	Blue
		4026	Violet
		3888	Violet
		3188	Violet
Hydrogen.........	Vacuum tube	6563*	Red
		4861†	Blue
		4341	Violet
		4102	Violet
Mercury...........	Mercury vapor lamp	5790	Yellow
		5770	Yellow
		5461	Green
		4358	Blue
Sodium...........	Bunsen flame.......	5896‡	Orange
		5890§	Orange

* Fraunhofer C Line.   ‡ Fraunhofer D_1 Line.
† Fraunhofer F Line.   § Fraunhofer D_2 Line.

TABLE IV

Optical Constants of Some Common Substances

Substance	n_D	n_C	n_F	ν
Carbon bisulphide................	1.6293	1.6198	1.6541	18.3
Ethyl alcohol....................	1.3624	1.3606	1.3667	59.4
"Average" crown glass...........	1.5116	1.50923	1.51762	61.0
"Average" medium flint glass.....	1.6150	1.60998	1.62667	36.8

Note.—ν in this table $= \dfrac{n_D - 1}{n_F - n_C} = \dfrac{1}{d}$.

Where d is the dispersive power
and ν may be called the "dispersion number."

TABLE V

CONSTANTS OF OPTICAL-GLASS TYPES MANUFACTURED IN THE
UNITED STATES

Manufacturer	n_D	n_C	n_F	ν
BOROSILICATE CROWN				
A	1.5164	1.51423	1.52218	64.9
B	1.5244	1.5219	1.5301	64.0
C	1.51579	1.51345	1.52152	63.8
ORDINARY CROWN				
A	1.5116	1.50923	1.51762	61.0
B	1.5198	1.51530	1.52380	60.9
C	1.52495	1.52170	1.53057	59.1
D	1.5209	1.5160	1.5246	60.6
DENSE BARIUM CROWN				
B	1.62149	1.61809	1.62968	53.6
C	1.61100	1.60796	1.61873	56.7
LIGHT FLINT				
A	1.5619	1.55879	1.55829	44.8
B	1.58484	1.58070	1.59509	40.5
C	1.5722	1.56817	1.58182	42.1
MEDIUM FLINT				
A	1.6150	1.60998	1.62667	36.8
B	1.6274	1.6225	1.6397	36.4
C	1.6180	1.61321	1.63007	36.6
D	1.6256	1.6203	1.6378	36.7
DENSE FLINT				
A	1.6465	1.64095	1.65999	33.9
B	1.65555	1.65019	1.66922	34.4
C	1.63500	1.62982	1.6480	34.9
EXTRA DENSE FLINT				
E	1.75614	1.74839	1.77617	27.2

Note.—ν in this table $= \dfrac{n_D - 1}{n_F - n_C} = \dfrac{1}{d}$.

Where d is the dispersive power
and ν may be called the "dispersion number."

TABLE VI

Convenient Sources of Ultra-Violet Radiation

1. Quartz-tube mercury-vapor lamp
2. Iron arc
3. Aluminum spark under water
4. Sun

TABLE VII

Wave Lengths in Ångströms of the Principal Lines in the Ultra-Violet Spectrum of the Quartz-Tube Mercury-Vapor Lamp

4360	3650	3130	2800	2550
4050	3340	3020	2650	2400

TABLE VIII

A Short List of Text-Books in Geometrical and Physical Optics

Name of Book	Author	Publishers
"Light for Students"	Edser	The Macmillan Co.
"A Treatise on Light"	Houstoun	Longmans, Green & Co.
"Physical Optics"	Wood	The Macmillan Co.
"Theory of Optics"	Drude	Longmans, Green & Co.
"The Theory of Optics"	Schuster	Edward Arnold, London
"The Principles of Physical Optics"	Mach	E. P. Dutton & Co.
"The Theory of Modern Optical Instruments"	Gleichen	His Majesty's Stationery Office
"The Theory of Optical Instruments"	Whittaker	Cambridge University Press
"Mirrors, Prisms, Lenses"	Southall	The Macmillan Co.
"Light Waves and Their Uses"	Michelson	University of Chicago Press
"Spectroscopy"	Baly	Longmans, Green & Co.
"Ultra-Violet Radiation"	Luckiesh	D. Van Nostrand & Co.
"Elementary Treatise on Geometrical Optics"	Heath	Cambridge University Press
"Elementary Geometrical and Physical Optics"	Wagner	In Preparation

TABLE IX

Four Place Logarithms

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	16	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	4	5	7	9	10	12	14	16
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	5	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	7	8	9	11	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	4	5	6	7	8	9	11
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	6	7	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	5	6	7	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

The proportional parts are stated in full for every tenth at the right-hand side. The logarithm of any number of four significant figures can be read directly by adding the proportional part corresponding to the fourth figure to the tabular number corresponding to the first three figures. There may be an error of 1 in the last place.

TABLE IX—*Continued*

Four Place Logarithms

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	1	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	3	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	5	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	3	3	4	4	5	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	3	3	4	4	5	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	4	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	1	1	2	2	3	3	4	4	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	3	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

The proportional parts are stated in full for every tenth at the right-hand side. The logarithm of any number of four significant figures can be read directly by adding the proportional part corresponding to the fourth figure to the tabular number corresponding to the first three figures. There may be an error of 1 in the last place.

TABLE X

Natural Trigonometric Functions

Rad	Deg	Tan	Sin	Log Sin	Log Cos	Cos	Cot		
0000	0	0000	0000	$-\infty$	0	1	8	90	$\pi/2$
0175	1	0175	0175	2419	9999	9998	57.29	89	1.553
0349	2	0349	0349	5428	9997	9994	28.64	88	1.536
0524	3	0524	0523	7188	9994	9986	19.08	87	1.518
0698	4	0699	0698	8436	9989	9976	14.30	86	1.501
0873	5	0875	0872	9403	9983	9962	11.43	85	1.484
1047	6	1051	1045	0192	9976	9945	9.514	84	1.466
1222	7	1228	1219	0859	9968	9925	8.144	83	1.449
1396	8	1405	1392	1436	9958	9903	7.115	82	1.431
1571	9	1584	1564	1943	9946	9877	6.314	81	1.414
1745	10	1763	1736	2397	9934	9848	5.671	80	1.396
1920	11	1944	1908	2806	9919	9816	5.145	79	1.379
2094	12	2126	2079	3179	9904	9781	4.705	78	1.361
2269	13	2309	2250	3521	9887	9744	4.331	77	1.344
2443	14	2493	2419	3837	9869	9703	4.011	76	1.326
2618	15	2679	2588	4130	9849	9659	3.732	75	1.309
2793	16	2867	2756	4403	9828	9613	3.487	74	1.292
2967	17	3057	2924	4659	9806	9563	3.271	73	1.274
3142	18	3249	3090	4900	9782	9511	3.078	72	1.257
3316	19	3443	3256	5126	9757	9455	2.904	71	1.239
3491	20	3640	3420	5341	9730	9397	2.747	70	1.222
3665	21	3839	3584	5543	9702	9336	2.605	69	1.204
3840	22	4040	3746	5736	9672	9272	2.475	68	1.187
4014	23	4245	3907	5919	9640	9205	2.356	67	1.169
4189	24	4452	4067	6093	9607	9135	2.246	66	1.152
4363	25	4663	4226	6259	9573	9063	2.145	65	1.134
4538	26	4877	4384	6418	9537	8988	2.050	64	1.117
4712	27	5095	4540	6570	9499	8910	1.963	63	1.100
4887	28	5317	4695	6716	9459	8829	1.881	62	1.082
5061	29	5543	4848	6856	9418	8746	1.804	61	1.065
5236	30	5774	5000	6990	9375	8660	1.732	60	1.047
5411	31	6009	5150	7118	9331	8572	1.664	59	1.030
5585	32	6249	5299	7242	9284	8480	1.600	58	1.012
5760	33	6494	5446	7361	9236	8387	1.540	57	9948
5934	34	6745	5592	7476	9186	8290	1.483	56	9774
6109	35	7002	5736	7586	9134	8192	1.428	55	9599
6283	36	7265	5878	7692	9080	8090	1.376	54	9425
6458	37	7536	6018	7795	9023	7986	1.327	53	9250
6632	38	7813	6157	7893	8965	7880	1.280	52	9076
6807	39	8098	6293	7989	8905	7771	1.235	51	8901
6981	40	8391	6428	8081	8843	7660	1.192	50	8727
7156	41	8693	6561	8169	8778	7547	1.150	49	8552
7330	42	9004	6691	8255	8711	7431	1.111	48	8378
7505	43	9325	6820	8338	8641	7314	1.072	47	8203
7679	44	9657	6947	8418	8569	7193	1.036	46	8029
7854	45	1	7071	8495	8495	7071	1.000	45	7854
		Cot	Cos	Log Cos	Log Sin	Sin	Tan	Deg	Rad

TABLE XI

Sin² Values

$\theta°$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002
1	0.0003	0.0004	0.0004	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0011
2	0.0012	0.0013	0.0015	0.0016	0.0018	0.0019	0.0021	0.0022	0.0024	0.0026
3	0.0027	0.0029	0.0031	0.0033	0.0035	0.0037	0.0039	0.0042	0.0044	0.0046
4	0.0049	0.0051	0.0054	0.0056	0.0059	0.0062	0.0064	0.0067	0.0070	0.0073
5	0.0076	0.0079	0.0082	0.0085	0.0089	0.0092	0.0095	0.0099	0.0102	0.0106
6	0.0109	0.0113	0.0117	0.0120	0.0124	0.0128	0.0132	0.0136	0.0140	0.0144
7	0.0149	0.0153	0.0157	0.0161	0.0166	0.0170	0.0175	0.0180	0.0184	0.0189
8	0.0194	0.0199	0.0203	0.0208	0.0213	0.0218	0.0224	0.0229	0.0234	0.0239
9	0.0245	0.0250	0.0256	0.0261	0.0267	0.0272	0.0278	0.0284	0.0290	0.0296
10	0.0302	0.0307	0.0314	0.0320	0.0326	0.0332	0.0339	0.0345	0.0351	0.0358
11	0.0364	0.0371	0.0377	0.0384	0.0391	0.0398	0.0404	0.0411	0.0418	0.0425
12	0.0432	0.0439	0.0447	0.0454	0.0461	0.0469	0.0476	0.0483	0.0491	0.0499
13	0.0506	0.0514	0.0522	0.0529	0.0537	0.0545	0.0553	0.0561	0.0569	0.0577
14	0.0585	0.0594	0.0602	0.0610	0.0619	0.0627	0.0635	0.0644	0.0653	0.0662
15	0.0670	0.0679	0.0687	0.0696	0.0705	0.0714	0.0723	0.0732	0.0741	0.0751
16	0.0760	0.0769	0.0778	0.0788	0.0797	0.0807	0.0816	0.0826	0.0835	0.0845
17	0.0855	0.0864	0.0875	0.0884	0.0894	0.0904	0.0914	0.0924	0.0935	0.0945
18	0.0955	0.0965	0.0975	0.0986	0.0997	0.1007	0.1017	0.1028	0.1039	0.1040
19	0.1060	0.1071	0.1081	0.1092	0.1103	0.1115	0.1125	0.1137	0.1148	0.1159
20	0.1170	0.1181	0.1192	0.1203	0.1215	0.1226	0.1238	0.1250	0.1261	0.1272
21	0.1284	0.1296	0.1308	0.1320	0.1331	0.1344	0.1355	0.1367	0.1379	0.1391
22	0.1404	0.1415	0.1428	0.1440	0.1452	0.1464	0.1477	0.1490	0.1502	0.1514
23	0.1527	0.1540	0.1552	0.1565	0.1578	0.1590	0.1603	0.1616	0.1629	0.1641
24	0.1654	0.1667	0.1680	0.1694	0.1707	0.1720	0.1733	0.1746	0.1760	0.1773
25	0.1786	0.1800	0.1813	0.1826	0.1840	0.1854	0.1867	0.1880	0.1894	0.1908
26	0.1921	0.1936	0.1949	0.1963	0.1977	0.1991	0.2005	0.2020	0.2033	0.2047
27	0.2061	0.2075	0.2089	0.2104	0.2117	0.2132	0.2147	0.2161	0.2175	0.2190
28	0.2204	0.2218	0.2233	0.2248	0.2263	0.2277	0.2292	0.2306	0.2321	0.2336
29	0.2351	0.2365	0.2380	0.2394	0.2410	0.2424	0.2440	0.2455	0.2469	0.2485
30	0.2500	0.2515	0.2530	0.2546	0.2561	0.2576	0.2592	0.2606	0.2622	0.2638
31	0.2652	0.2668	0.2684	0.2699	0.2714	0.2730	0.2745	0.2760	0.2777	0.2793
32	0.2809	0.2824	0.2839	0.2855	0.2871	0.2887	0.2903	0.2919	0.2935	0.2950
33	0.2966	0.2983	0.2998	0.3015	0.3030	0.3046	0.3063	0.3079	0.3095	0.3110
34	0.3128	0.3143	0.3159	0.3175	0.3192	0.3208	0.3224	0.3240	0.3257	0.3272
35	0.3290	0.3307	0.3322	0.3339	0.3356	0.3373	0.3389	0.3406	0.3421	0.3439
36	0.3455	0.3472	0.3488	0.3504	0.3522	0.3538	0.3552	0.3571	0.3588	0.3606
37	0.3622	0.3639	0.3656	0.3673	0.3690	0.3705	0.3722	0.3739	0.3757	0.3774
38	0.3790	0.3807	0.3825	0.3841	0.3858	0.3874	0.3892	0.3908	0.3926	0.3943
39	0.3961	0.3978	0.3994	0.4012	0.4029	0.4046	0.4063	0.4079	0.4098	0.4115
40	0.4133	0.4150	0.4167	0.4184	0.4202	0.4217	0.4235	0.4252	0.4270	0.4288
41	0.4303	0.4321	0.4339	0.4355	0.4373	0.4391	0.4408	0.4426	0.4442	0.4461
42	0.4477	0.4496	0.4512	0.4529	0.4548	0.4565	0.4582	0.4598	0.4617	0.4634
43	0.4651	0.4669	0.4686	0.4703	0.4721	0.4738	0.4756	0.4773	0.4791	0.4808
44	0.4826	0.4844	0.4860	0.4878	0.4896	0.4914	0.4930	0.4948	0.4966	0.4982

TABLE XI—*Continued*

Sin² Values

$\theta°$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
45	0.5000	0.5017	0.5035	0.5051	0.5070	0.5086	0.5105	0.5122	0.5141	0.5157
46	0.5174	0.5193	0.5210	0.5226	0.5243	0.5263	0.5280	0.5297	0.5314	0.5331
47	0.5351	0.5365	0.5383	0.5400	0.5418	0.5435	0.5453	0.5470	0.5488	0.5506
48	0.5523	0.5541	0.5557	0.5575	0.5593	0.5611	0.5626	0.5644	0.5663	0.5678
49	0.5696	0.5712	0.5731	0.5746	0.5765	0.5781	0.5800	0.5816	0.5834	0.5851
50	0.5869	0.5886	0.5902	0.5921	0.5937	0.5954	0.5970	0.5990	0.6006	0.6023
51	0.6046	0.6056	0.6073	0.6090	0.6107	0.6124	0.6140	0.6157	0.6174	0.6192
52	0.6209	0.6226	0.6243	0.6260	0.6280	0.6295	0.6310	0.6327	0.6345	0.6362
53	0.6377	0.6394	0.6412	0.6430	0.6445	0.6462	0.6477	0.6495	0.6513	0.6528
54	0.6546	0.6561	0.6580	0.6595	0.6610	0.6628	0.6644	0.6662	0.6677	0.6693
55	0.6710	0.6727	0.6742	0.6759	0.6775	0.6792	0.6808	0.6823	0.6841	0.6856
56	0.6872	0.6890	0.6906	0.6922	0.6937	0.6953	0.6970	0.6986	0.7002	0.7018
57	0.7034	0.7050	0.7065	0.7081	0.7097	0.7114	0.7129	0.7145	0.7160	0.7176
58	0.7191	0.7208	0.7223	0.7238	0.7254	0.7270	0.7284	0.7301	0.7316	0.7332
59	0.7347	0.7362	0.7377	0.7393	0.7408	0.7423	0.7439	0.7454	0.7470	0.7485
60	0.7501	0.7514	0.7530	0.7544	0.7560	0.7575	0.7589	0.7605	0.7621	0.7635
61	0.7649	0.7665	0.7679	0.7693	0.7707	0.7723	0.7737	0.7752	0.7768	0.7782
62	0.7796	0.7811	0.7825	0.7840	0.7854	0.7869	0.7881	0.7896	0.7910	0.7925
63	0.7940	0.7952	0.7967	0.7982	0.7995	0.8009	0.8022	0.8037	0.8050	0.8065
64	0.8078	0.8093	0.8106	0.8119	0.8134	0.8147	0.8160	0.8173	0.8187	0.8200
65	0.8215	0.8228	0.8241	0.8255	0.8268	0.8279	0.8293	0.8306	0.8320	0.8333
66	0.8346	0.8360	0.8372	0.8385	0.8397	0.8410	0.8424	0.8435	0.8449	0.8461
67	0.8474	0.8486	0.8498	0.8511	0.8523	0.8535	0.8549	0.8561	0.8572	0.8584
68	0.8596	0.8608	0.8622	0.8634	0.8646	0.8658	0.8670	0.8680	0.8692	0.8704
69	0.8716	0.8728	0.8740	0.8750	0.8762	0.8774	0.8784	0.8796	0.8808	0.8819
70	0.8831	0.8841	0.8853	0.8863	0.8876	0.8886	0.8896	0.8908	0.8919	0.8929
71	0.8939	0.8952	0.8962	0.8972	0.8983	0.8993	0.9003	0.9014	0.9024	0.9034
72	0.9045	0.9055	0.9066	0.9076	0.9087	0.9095	0.9105	0.9116	0.9126	0.9135
73	0.9145	0.9156	0.9164	0.9175	0.9183	0.9194	0.9203	0.9213	0.9221	0.9230
74	0.9241	0.9249	0.9258	0.9268	0.9277	0.9285	0.9294	0.9305	0.9313	0.9322
75	0.9330	0.9339	0.9348	0.9356	0.9365	0.9374	0.9382	0.9391	0.9397	0.9406
76	0.9415	0.9423	0.9432	0.9438	0.9447	0.9456	0.9463	0.9471	0.9478	0.9486
77	0.9493	0.9502	0.9508	0.9517	0.9524	0.9532	0.9539	0.9546	0.9554	0.9561
78	0.9568	0.9574	0.9581	0.9590	0.9596	0.9603	0.9609	0.9616	0.9623	0.9629
79	0.9636	0.9643	0.9650	0.9656	0.9660	0.9667	0.9674	0.9681	0.9687	0.9692
80	0.9698	0.9704	0.9710	0.9716	0.9722	0.9728	0.9733	0.9739	0.9744	0.9750
81	0.9755	0.9761	0.9766	0.9771	0.9777	0.9782	0.9787	0.9792	0.9797	0.9801
82	0.9806	0.9811	0.9816	0.9820	0.9825	0.9830	0.9834	0.9838	0.9843	0.9847
83	0.9852	0.9856	0.9860	0.9864	0.9868	0.9872	0.9876	0.9880	0.9883	0.9887
84	0.9891	0.9894	0.9898	0.9901	0.9905	0.9908	0.9912	0.9915	0.9918	0.9921
85	0.9924	0.9927	0.9930	0.9933	0.9936	0.9938	0.9941	0.9944	0.9946	0.9949
86	0.9951	0.9954	0.9956	0.9958	0.9960	0.9963	0.9965	0.9967	0.9969	0.9971
87	0.9973	0.9974	0.9976	0.9978	0.9979	0.9981	0.9983	0.9984	0.9985	0.9987
88	0.9988	0.9989	0.9990	0.9991	0.9992	0.9993	0.9994	0.9995	0.9996	0.9996
89	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000

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